KOMBINASI DUA DISTRIBUSI WEIBULL DENGAN MENGGUNAKAN PARAMETER CAMPURAN

TUGAS AKHIR

Diajukan Sebagai Salah Satu Syarat
Untuk Memperoleh Gelar Sarjana Teknik Pada
Jurusan Matematika

Oleh:

FIVI SRI NINGSIH
106 5400 4473

FAKULTAS SAINS DAN TEKNOLOGI
UNIVERSITAS ISLAM NEGRI SULTAN SYARIF KASIM RIAU
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FIVI SRI NINGSIH
10654004473

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Jurusan Matematika
Fakultas Sains dan Teknologi
Universitas Islam Negeri Sultan Syarif Kasim Riau
Jl. Soebrantas No. 155 Pekanbaru

ABSTRAK

Penelitian ini akan membahas tentang kombinasi dua distribusi Weibull dengan menggunakan parameter campuran. Dua distribusi Weibull itu adalah distribusi Weibull dua parameter dan tiga parameter dengan parameter campuran adalah w, selanjutnya akan dicari nilai masing-masing parameter dengan menggunakan estimasi maksimum *likelihood*. Hasil kombinasi memiliki bentuk densitas peluang sebagai berikut:

$$f(x) \quad = w\left(\alpha_1 \frac{x^{\alpha_1 - 1}}{\beta_1^{\alpha_1}}\right).e^{-\left(x/\beta_1\right)^{\alpha_1}} + (1-w)\left(\frac{\alpha_2 (x-\gamma)^{\alpha_2 - 1}}{\beta_2^{\alpha_2}}\right).e^{-\left(x/\beta_2\right)^{\alpha_2}}$$

pada penelitian ini juga diaplikasikan ke dalam *lifetimes of electronic components* dengan mengambil nilai $w = 0.4$ dan $\gamma = 1$ sampai iterasi kedua. Sehingga diperoleh nilai masing-masing parameter sebagai berikut:

$$\alpha_1^2 = 0.120, \beta_1^2 = -0.057, \alpha_2^2 = 2.411, \beta_2^2 = 0.125, \gamma = 0.799, w = 0.95$$

Kata kunci: Distribusi Weibull, Estimasi Maksimum *Likelihood*, Newton Rapson, Nilai awal.

COMBINING TWO WEIBULL DISTRIBUTIONS USING A MIXING PARAMETER

FIVI SRI NINGSIH
10654004473

Date of Final Exam : June, 28th 2010
Graduation Cremony Priod: October, 2010

Matematich Engineering Departement
Faculty of Sciences and Technology
State Islamic University of Sultan Syarif Kasim Riau
Soebrantas Street No. 155 Pekanbaru

ABSRACT

This thesis will discuss about the combination of two Weibull distributions using a mixing parameter. The two Weibull distributions are two and three parameters by mixed parameters w*. Next will search for each parameter using estimation maximum likelihood. The result combination has foam of density:*

$$f(x) \;=\; w\left(\alpha_1 \frac{x^{\alpha_1 - 1}}{\beta_1^{\alpha_1}}\right).e^{-\left(x/\beta_1\right)^{\alpha_1}} + (1-w)\left(\frac{\alpha_2 (x-\gamma)^{\alpha_2 - 1}}{\beta_2^{\alpha_2}}\right).e^{-\left(x/\beta_2\right)^{\alpha_2}}$$

the research has aplications in to lifetimes of electronic components by got score $w = 0.4$ *and* $\gamma = 1$ *until second interation. Than could get value for each parameters are:*

$$\alpha_1^2 = 0.120, \beta_1^2 = -0.057, \alpha_2^2 = 2.411, \beta_2^2 = 0.125, \gamma = 0.799, w = 0.95$$

Keyword: *Weibull Distribution, Estimations Maksimum Likelihood, Newton Raphson, First Score.*

DAFTAR ISI

BAB I
PENDAHULUAN

1.1 Latar Belakang

Seiring dengan perkembangan ilmu matematika, para ilmuan terus mengembangkan teori-teori yang telah ada seperti konsep distribusi, distribusi diskrit dan distribusi kontinu. Distribusi kontinu meliputi distribusi eksponensial, distribusi normal, distribusi Weibull dan lain-lain.

Distribusi Weibull pertama kali diperkenalkan oleh ahli Fisika dari Swedia Waloddy Weibull pada tahun 1939. Dalam aplikasinya, distribusi ini sering digunakan untuk memodelkan waktu sampai kegagalan (*time to failure*) dari suatu sistem fisika. Adapun distribusi Weibull dua parameter memiliki parameter α (*shape parameter*), β (*scale parameter*) yang memiliki bentuk umum sebagai berikut:

$$f(x:\alpha,\beta)=\frac{\alpha}{\beta^{\alpha}}x^{\alpha-1}e^{-\left(x/\beta\right)^{\alpha}}$$

distribusi Weibull tiga parameter memiliki α,β dan γ (*location parameter*), distribusi weibull ini memiliki bentuk umum sebagai berikut:

$$f(x:\alpha,\beta,\gamma)=\frac{\alpha(x-\gamma)^{\alpha-1}}{\beta^{\alpha}}e^{-(x-\gamma)^{\alpha}/\beta^{\alpha}}$$

Berdasarkan jurnal "*Combining Two Weibull Distribution Using a Mixing Parameter*" oleh Ahmad Mahir Razali dan Ali Salih (2009) yang membahas tentang dua buah distribusi Weibull yang dikombinasikan dengan menggunakan parameter campuran, masing-masing parameter campuran itu adala w dan $(1-w)$ yang mana $\sum w_i=1$. Hasil kombinasi ini disebut dengan distribusi Weibull campuran. Estimasi parameter distribusi Weibull campuran ini menggunakan estimasi maksimum *likelihood* untuk menentukan nilai awal dari suatu parameter. Berdasarkan uraian di atas penulis tertarik untuk mengajukan judul *"Kombinasi Dua Distribusi Weibull dengan Menggunakan Parameter Campuran"*.

1.2 Rumusan Masalah

Berdasarkan uraian latar belakang, maka permasalahan yang akan dibahas adalah bagaimana mengkombinasikan dua distribusi Weibull dengan menggunakan parameter campuran.

1.3 Batasan Masalah

Batasan masalah pada proposal tugas akhir ini adalah mengkombinasi dua distribusi Weibull menggunakan parameter campuran, selanjutnya akan diaplikasikan dengan menggunakan data *lifetime of electronic components*.

1.4 Tujuan

Tujuan yang ingin dicapai adalah mengkombinasikan dua distribusi Weibull menggunakan parameter campuran sehingga menghasilkan Weibull campuran, selanjutnya akan dicari nilai masing-masing parameter campuran menggunakan estimasi maksimum *likelihood.*

1.5 Manfaat

Distribusi Weibull campuran sering diaplikasikan dalam memodelkan "waktu sampai kegagalan (*time to failure*)" dari suatu sistem Fisika. Misalnya, pada sistem dimana jumlah kegagalan meningkat dengan berjalannya waktu (contoh alat elektronik).

1.6 Sistematika Penulisan

Adapun sistematika penulisan tugas akhir ini terdiri dari beberapa bab, yang memberikan gambaran secara menyeluruh, yaitu:

BAB I PENDAHULUAN

Bab ini berisikan tentang deskripsi umum isi tugas akhir yang meliputi latar belakang masalah, rumusan masalah, batasan masalah, tujuan penyusunan dan sistematika penulisan.

BAB II LANDASAN TEORI

Bab ini berisikan mengenai penjelasan dasar teori yang mendukung dalam penyelesaian tugas akhir.

BAB III METODOLOGI

Bab ini memuat uraian tentang distribusi Weibull dua parameter dan tiga parameter serta karakteristiknya. Kemudian dilanjutkan dengan mengkombinasikan kedua distribusi menggunakan parameter campuran.

BAB IV ISI

Bab ini berisikan penjabaran kombinasi dua distribusi Weibull menggunakan parameter campuran. Selanjutnya menggunakan metode maximum *likelihood* untuk menentukan nilai awal distribusi Weibull campuran.

BAB V PENUTUP

Bab ini berisikan kesimpulan dari tugas akhir dan saran-saran penulis kepada pembaca agar distribusi Weibull ini dapat dikembangkan lagi.

BAB II
LANDASAN TEORI

Secara umum dalam statistika dikenal dua macam distribusi peluang yaitu distribusi peluang dengan variabel acak diskrit dan distribusi peluang variable acak kontinu. Distribusi peluang variabel acak diskrit dapat kita asumsikan hanya terbatas atau tak terbatas dapat dihitung dengan jumlah yang jelas, sedangkan distribusi peluang untuk variabel kontinu tidak memiliki nilai yang pasti.

2.1 Variabel Acak Kontinu dan Ekspektasi Acak Kontinu.

Variabel acak yang dibahas dalam tugas akhir ini adalah variabel acak kontinu yang dinotasikan dengan x. Bila x adalah suatu fungsi yang memasangkan setiap elemen t dalam ruang sampel S tepat kepada satu bilangan riil, maka $x(t)$ disebut variabel acak.

Fungsi densitas peluang variabel acak kontinu dengan ruang yaitu interval atau gabungan interval adalah fungsi $f(x)$ yang diintegralkan yang memenuhi kondisi :

1. $f(x) > 0$
2. $\int_{-\infty}^{\infty} f(x)\,dx = 1$

Definisi 2.1 (Dennis dkk, 2002) Misalkan x adalah variabel acak yang mempunyai fungsi densitas $f(x)$, maka ekspektasi dari x yang dinotasikan dengan $E(X)$ didefenisikan :

$$E(X) = \int_{-\infty}^{\infty} x.\,f(x)\,dx \tag{2.1}$$

$E(X)$ disebut juga sebagai nilai rata-rata dari x. Sedangkan variasi dari x dapat ditentukan berdasarkan perumusan secara ekspektasi, yaitu :

$$Var(X) = E(X^2) - (E(X))^2 \tag{2.2}$$

2.2 Distribusi Weibull

Distribusi Weibull sering diaplikasikan untuk memodelkan “waktu sampai kegagalan (*time to failure*)” dari suatu sistem Fisika. Misalnya, yaitu pada sistem yang mana jumlah kegagalan meningkat dengan berjalannya waktu (misalnya alat elektronik).

Definisi 2.2 (William dkk, 1990) Suatu variabel acak kontinu X dikatakan berdistribusi Weibull dengan parameter bentuk α dan faktor skala β, dimana $\alpha > 0$ dan $\beta > 0$, maka fungsi densitas dari X adalah :

$$f(x;\alpha,\beta) = \begin{cases} \frac{\alpha}{\beta^{\beta}} x^{\alpha-1} e^{-(x/\beta)^{\alpha}} & x \geq 0 \\ 0 & untuk\ yang\ lain \end{cases} \tag{2.3}$$

sedangkan distribusi Weibull tiga parameter memiliki bentuk umum fungsi densitas sebagai berikut:

$$f(x;\alpha,\beta) = \begin{cases} \frac{\alpha}{\beta^{\beta}} (x-\gamma)^{\alpha-1} e^{-((x-\gamma)/\beta)^{\alpha}} & x \geq \gamma \\ 0 & untuk\ yang\ lain \end{cases} \tag{2.4}$$

Definisi 2.3 (Harinaldi, 2005) Fungsi distribusi komulatif variabel X dinotasikan sebagai F_x dan didefinisikan sebagai $F_x(x) = p(X \leq x)$ untuk seluruh x yang riil. Jika X adalah kontinu, maka:

$$F_x(x) = \int_{-\infty}^{x} f(t)\ dt \tag{2.5}$$

Definisi 2.4 (Lee dkk, 2003) Misal T adalah waktu survival, dari distribusi T dapat ditentukan fungsi survival (*surving function*), yang dinotasikan oleh $s(t)$. Fungsi survival atau fungsi selamat dapat didefenisikan sebagai peluang kesuksesan lebih lama dari *t*, sehingga dapat ditulis :

$$s(t) = 1 - F(t) \tag{2.6}$$

Definisi 2.5 (Lee dkk, 2003) Fungsi bahaya atau dinotasikan dengan $h(t)$ dengan waktu survival T yang memberikan nilai kegagalan bersyarat, estimasi fungsi berbahaya diberikan oleh:

$$h(t) = \frac{f(t)}{1 - F(t)} \tag{2.7}$$

dengan $f(t)$ = fungsi densitas peluang

$F(t)$ = fungsi distribusi kumulatif

Fungsi Hazard (bahaya) kemungkinan bisa menaik, tetap konstan, menurun atau menunjukkan proses yang lebih kompleks.

Definisi 2.6 (Dennis dkk, 2002) Variabel acak X dikatakan memiliki distribusi gamma dengan parameter $\alpha > 0$ dan $\beta > 0$ jika dan hanya jika fungsi densitas dari Y adalah:

$$f(x) = \begin{cases} \dfrac{x^{\alpha-1} e^{-x/\beta}}{\beta^{\alpha}\Gamma(\alpha)} & 0 \le x < \infty \\ 0 & untuk\ lainnya \end{cases} \tag{2.8}$$

dengan:

$$\Gamma(\alpha) = \int_0^{\infty} x^{\alpha-1} e^{-x}\, dx \tag{2.9}$$

Kuantitas $\Gamma(\alpha)$ dikenal dengan fungsi gamma. Integral secara langsung akan menghasilkan $\Gamma(1) = 1$. Dan secara terus-menerus integral akan menghasilkan bahwa $\Gamma(\alpha) = (\alpha - 1)\Gamma(\alpha - 1)$ untuk $\alpha > 1$, dan juga $\Gamma(n) = (n-1)!$ yang dihasilkan jika n adalah bilangan bulat. Pembuktian persamaan (2.8) dapat ditunjukkan sebagai berikut:

$$\begin{aligned} \Gamma(\alpha) &= \int_0^{\infty} x^{\alpha-1} e^{-x}\, dx \\ &= \left[-x^{\alpha-1} e^{-x}\right]_0^{\infty} + \int_0^{\infty} (\alpha-1) x^{\alpha-2} e^{-x} dx \\ &= (\alpha-1)\int_0^{\infty} x^{\alpha-2} e^{-x}\, dx \\ &= (\alpha-1)\Gamma(\alpha-1) \end{aligned} \tag{2.10}$$

Fungsi densitas peluang gamma dapat memenuhi sifat distribusi peluang kontinu, dengan pembuktian persamaan sebagai berikut:

$$\int_{-\infty}^{\infty} f(x)\, dx = \int_0^{\infty} \frac{x^{\alpha-1} e^{-x/\beta}}{\Gamma(\alpha)\beta^{\alpha}} dx$$

misalkan:

$$y = \frac{x}{\beta}$$
$$x = y\beta$$
$$dx = \beta\, dy$$

maka:

$$= \int_0^\infty \frac{(y\beta)^{\alpha-1} e^{-y}}{\Gamma(\alpha)\beta^\alpha} \beta \, dy$$

$$= \frac{1}{\Gamma(\alpha)} \int_0^\infty y^{\alpha-1} e^{-y} \, dy = \frac{\Gamma(\alpha)}{\Gamma(\alpha)} = 1$$

Akan ditunjukkan apakah fungsi densitas pada persamaan (2.3) dan (2.4) memenuhi sifat distribusi kontinu, yaitu:

$$\int_{-\infty}^{\infty} f(x)\, dx = 1$$

maka akan ditunjukkan fungsi densitas pada persamaan (2.3) memenuhi sifat distribusi kontinu di atas, sebagai berikut:

$$\int_{-\infty}^{\infty} f(x)\, dx = 1$$

$$\int_0^\infty \frac{\alpha_1}{\beta_1^{\alpha_1}} x^{\alpha_1 - 1} e^{-(x/\beta_1)^{\alpha_1}} \, dx = 1$$

misalkan:

$$u = (x/\beta_1)^{\alpha_1}$$

$$du = \frac{\alpha_1}{\beta_1} \left(x/\beta_1 \right)^{\alpha_1 - 1} dx$$

$$\frac{\beta_1}{\alpha_1} \left(\beta_1/x \right)^{\alpha_1 - 1} du = dx$$

maka:

$$\int_0^\infty \frac{\alpha_1}{\beta_1^{\alpha_1}} x^{\alpha-1} e^{-u} \frac{\beta_1}{\alpha_1} \left(\beta_1/x \right)^{\alpha_1 - 1} du = 1$$

$$\int_0^\infty e^{-u} \, du = 1$$

$$-e^{-u} \Big|_0^\infty = 1$$

$$0 - (-e^0) = 1$$

$$1 = 1$$

Selanjutnya akan ditunjukkan rata-rata distribusi Weibull dua parameter pada persamaan (2.3) berdasarkan persamaan (2.1), sebagai berikut:

$$E(X) = \int_{-\infty}^{\infty} x.f(x)\,dx$$

$$= \int_{-\infty}^{\infty} x.f(x)\,dx$$

$$= \int_{0}^{\infty} x.\frac{\alpha_1}{\beta_1^{\alpha_1}} x^{\alpha_1 - 1} e^{-(x/\beta_1)^{\alpha_1}} \quad dx$$

$$= \int_{0}^{\infty} \frac{\alpha_1}{\beta_1^{\alpha_1}} x^{\alpha_1} e^{-(x/\beta_1)^{\alpha_1}} \quad dx$$

misalkan:

$$\theta = \beta_1^{\alpha_1}$$

$$z = x^{\alpha_1}$$

$$z^{1/\alpha_1} = x$$

$$dx = \frac{1}{\alpha_1} z^{1/\alpha_1 - 1} dz$$

maka:

$$E(X) = \int_{0}^{\infty} \frac{\alpha_1}{\theta} z\, e^{-(z/\theta)} \;\; \frac{1}{\alpha_1} z^{1/\alpha_1 - 1} dz$$

$$= \int_{0}^{\infty} \frac{1}{\theta} e^{-(z/\theta)} z^{1/\alpha_1} dz$$

$$= \int_{0}^{\infty} \frac{1}{\theta} e^{-(z/\theta)} z^{1/\alpha_1} \frac{\theta^{1/\alpha_1 + 1}\Gamma(1/\alpha_1 + 1)}{\theta^{1/\alpha_1 + 1}\Gamma(1/\alpha_1 + 1)} dz$$

$$= \frac{\theta^{1/\alpha_1 + 1}\Gamma(1/\alpha_1 + 1)}{\theta} \int_{0}^{\infty} \frac{e^{-(z/\theta)} z^{1/\alpha_1}}{\theta^{1/\alpha_1 + 1}\Gamma(1/\alpha_1 + 1)} dz$$

dengan:

$$\int_{0}^{\infty} \frac{e^{-(z/\theta)} z^{1/\alpha_1}}{\theta^{1/\alpha_1 + 1}\Gamma(1/\alpha_1 + 1)} dz = 1$$

sehingga:

$$E(X)=\frac{\theta^{1/\alpha_1+1}\Gamma(1/\alpha_1+1)}{\theta}.1$$

$$=\frac{\theta^{1/\alpha_1+1}\Gamma(1/\alpha_1+1)}{\theta}$$

dengan $\theta=\beta_1^{\alpha_1}$

maka:

$$E(X)=\frac{\theta^{1/\alpha_1+1}\Gamma(1/\alpha_1+1)}{\theta}$$

$$=\beta_1\,\Gamma(1/\alpha_1+1) \qquad (2.11)$$

jadi diperoleh rata-rata distribusi Weibull dua parameter sebagai berikut:

$$E(X)=\beta_1\,\Gamma(1/\alpha_1+1)$$

Selanjutnya, variansi distribusi Weibull dapat diperoleh dari persamaan (2.2), yaitu:

$$V(X)=E(X^2)-(E(X))^2$$

Terlebih dahulu akan ditunjukkan:

$$E(X^2)=\int_{-\infty}^{\infty}x^2\ f(x)\,dx$$

$$=\int_0^{\infty}x^2.\frac{\alpha_1 x^{\alpha_1-1}}{\beta_1^{\alpha_1}}e^{-(x/\beta_1)^{\alpha_1}}\ dx$$

$$=\int_0^{\infty}\frac{\alpha_1 x^{\alpha_1+1}}{\beta_1^{\alpha_1}}e^{-x^{\alpha_1}/\beta^{\alpha_1}}\ dx$$

misalkan:

$$\theta=\beta_1^{\alpha_1}$$

$$z=x^{\alpha_1}$$

$$z^{1/\alpha_1}=x$$

$$dx=1/\alpha_1\ z^{\frac{1}{\alpha_1}-1}\ dz$$

sehingga:

$$E(X^2)=\int_0^{\infty}\frac{\alpha\, z.z^{1/\alpha}}{\theta}e^{-z/\theta}.\,{}^{1}\!/_{\alpha}\, z^{1/\alpha-1}\,dz$$

$$=\int_0^{\infty}\frac{z^{2/\alpha_1}}{\theta}\,e^{-z/\theta}\,dz$$

$$=\int_0^{\infty}\frac{z^{2/\alpha_1}}{\theta}e^{-z/\theta}\frac{\theta^{2/\alpha_1+1}}{\theta^{2/\alpha_1+1}}\frac{\Gamma(2/\alpha_1+1)}{\Gamma(2/\alpha_1+1)}dz$$

$$E(X^2)=\frac{\theta^{2/\alpha_1+1}\Gamma(2/\alpha_1+1)}{\theta}\int_0^{\infty}\frac{z^{2/\alpha_1}e^{-z/\theta}}{\theta^{2/\alpha_1+1}\Gamma(2/\alpha_1+1)}dz$$

$$=\frac{\theta^{\frac{2}{\alpha_1}+1}\,\Gamma(2/\alpha_1+1)}{\theta}$$

yang mana $\theta=\beta_1^{\alpha_1}$, sehingga:

$$E(x^2)=\beta_1^{\,2}\,\Gamma(2/\alpha_1+1) \tag{2.12}$$

Berdasarkan persamaan (2.11) dan (2.12), maka diperoleh variansi distribusi Weibull dengan menggunakan persamaan (2.2), yang akan ditunjukkan sebagai berikut:

$$V(X)=E(X^2)-((E(X))^2$$

$$=\beta_1^{\,2}\,\Gamma(2/\alpha_1+1)-((\beta_1\,\Gamma(1/\alpha_1+1))^2$$

$$=\beta_1^{\,2}(\Gamma(2/\alpha_1+1)-(\Gamma(1/\alpha_1+1))^2 \tag{2.13}$$

jadi diperoleh $V(X)=\beta_1^{\,2}(\Gamma(2/\alpha_1+1)-(\Gamma(1/\alpha_1+1))^2$

Selanjutnya akan ditunjukkan fungsi kumulatif dari persamaan (2.3) dengan menggunakan persamaan (2.5), sebagai berikut:

$$F(x)=\int_0^{x}\frac{\alpha_1 x^{\alpha_1-1}}{\beta_1^{\alpha_1}}e^{-(x/\beta_1)^{\alpha_1}}\,dx$$

misalkan:

$$u = \left(\frac{x}{\beta_1}\right)^{\alpha_1}$$

$$du = \alpha_1 \left(\frac{x}{\beta_1}\right)^{\alpha_1 - 1} . \frac{1}{\beta_1} dx$$

$$du = \frac{\alpha_1}{\beta_1} \left(\frac{x}{\beta_1}\right)^{\alpha_1 - 1} dx \text{ s}$$

$$\frac{\beta_1}{\alpha_1} \left(\frac{\beta_1}{x}\right)^{\alpha_1 - 1} du = dx$$

sehingga :

$$F(t) = \int_0^x \frac{\alpha_1}{\beta_1^{\alpha_1}} . x^{\alpha_1 - 1} . e^{-u} . \frac{\beta_1}{\alpha_1} \left(\frac{\beta_1}{x}\right)^{\alpha_1 - 1} du$$

$$= \int_0^x e^{-u} du$$

$$= -e^{-u} \Big|_0^x$$

$$= -e^{-\left(x/\beta_1\right)^{\alpha_1}} \Big|_0^x$$

$$= -e^{-\left(x/\beta_1\right)^{\alpha_1}} - \left(-e^{-\left(0/\beta_1\right)^{\alpha_1}}\right)$$

$$= -e^{-\left(x/\beta_1\right)^{\alpha_1}} + 1$$

$$F(t) = 1 - e^{-\left(x/\beta_1\right)^{\alpha_1}} \tag{2.14}$$

Selanjutnya fungsi survival diperoleh dari persamaan (2.6), yaitu:

$$s(t) = 1 - F(t)$$

$$= 1 - \left(1 - e^{-\left(x/\beta_1\right)^{\alpha_1}}\right)$$

$$= e^{-\left(x/\beta_1\right)^{\alpha_1}} \tag{2.15}$$

fungsi Hazard diperoleh dari persamaan (2.7), yaitu:

$$h(t) = \frac{f(t)}{1-F(t)}$$

$$= \frac{\dfrac{\alpha_1 x^{\alpha_1 -1}}{\beta_1^{\alpha_1}} e^{-(x/\beta_1)^{\alpha_1}}}{e^{-(x/\beta_1)^{\alpha_1}}}$$

$$= \frac{\alpha_1 x^{\alpha_1 -1}}{\beta_1^{\alpha_1}} \quad \blacksquare \tag{2.16}$$

Distribusi Weibull tiga parameter memiliki parameter α_2 (*shape parameter*), β_2 (*scale parameter*), γ (*location parameter*). Distribusi ini memiliki fungsi densitas peluang seperti pada persamaan (2.4), maka akan ditunjukkan bahwa :

$$\int_{-\infty}^{\infty} f(x)\, dx = 1$$

Bukti:

$$\int_{-\infty}^{\infty} f(x)\, dx = 1$$

$$\int_{\gamma}^{\infty} \frac{\alpha_2 (x-\gamma)^{\alpha_2 -1}}{\beta_2^{\alpha_2}} e^{-(x-\gamma)^{\alpha_2}/\beta_2^{\alpha_2}}\, dx = 1$$

misalkan:

$$u = \left((x-\gamma)/\beta_2 \right)^{\alpha_2}$$

$$du = \frac{\alpha_2}{\beta_2} \left((x-\gamma)/\beta_2 \right)^{\alpha_2 -1} dx \Rightarrow \frac{\beta_2}{\alpha_2} \left(\beta_2/(x-\gamma) \right)^{\alpha_2 -1} du = dx$$

maka:

$$\int_{\gamma}^{\infty} \frac{\alpha_2 (x-\gamma)^{\alpha_2 -1}}{\beta_2^{\alpha_2}} e^{-u} \frac{\beta_2}{\alpha_2} \left(\beta_2/(x-\gamma) \right)^{\alpha_2 -1} du = 1$$

$$\int_{\gamma}^{\infty} e^{-u} du = 1$$

$$-e^{-u} \Big|_{\gamma}^{\infty} = 1$$

$$-e^{-(x-\gamma)^{\alpha_2}/\beta_2^{\alpha_2}} \Big|_{\gamma}^{\infty} = 1$$

$$0 - (-e^{0}) = 1$$

$$1 = 1$$

Fungsi komulatif dari persamaan (2.4) dengan menggunakan persamaan (2.5), ditunjukkan sebagai berikut:

$$F(x) = \int_{\gamma}^{x} \frac{\alpha_2 (x-\gamma)^{\alpha_2 - 1}}{\beta_2^{\alpha_2}} e^{-(x-\gamma)^{\alpha_2} / \beta_2^{\alpha_2}} dx$$

misalkan:

$$u = \left(\frac{(x-\gamma)}{\beta_2} \right)^{\alpha_2}$$

$$du = \alpha_2 \left(\frac{(x-\gamma)}{\beta_2} \right)^{\alpha_2 - 1} \frac{1}{\beta_2} dx$$

$$\frac{\beta_2}{\alpha_2} \left(\frac{\beta_2}{(x-\gamma)} \right)^{\alpha_2 - 1} du = dx$$

maka:

$$F(x) = \int_{\gamma}^{x} \frac{\alpha_2 (x-\gamma)^{\alpha - 1}}{\beta_2^{\alpha_2}} e^{-u} \frac{\beta_2}{\alpha_2} \left(\frac{\beta_2}{(x-\gamma)} \right)^{\alpha_2 - 1} du$$

$$= \int_{\gamma}^{x} e^{-u} du$$

$$= -e^{-u} \Big|_{\gamma}^{x}$$

$$= -e^{-(x-\gamma)^{\alpha_2} / \beta_2^{\alpha_2}} \Big|_{\gamma}^{x}$$

$$= -e^{-(x-\gamma)^{\alpha_2} / \beta_2^{\alpha_2}} - (-1)$$

$$= 1 - e^{-(x-\gamma)^{\alpha_2} / \beta_2^{\alpha_2}} \qquad (2.17)$$

Selanjutnya fungsi survival dari persamaan (2.4) dapat diperoleh dari persamaan (2.6) sebagai berikut:

$$s(t) = 1 - F(t)$$

$$= 1 - (1 - e^{-(x-\gamma)^{\alpha_2} / \beta_2^{\alpha_2}})$$

$$= e^{-(x-\gamma)^{\alpha_2} / \beta_2^{\alpha_2}} \qquad (2.18)$$

Fungsi Hazard dari persamaan (2.4) diperoleh dari persamaan (2.7), yaitu:

$$h(t) = \frac{f(t)}{1 - F(t)}$$

$$= \frac{\dfrac{\alpha_2 (x-\gamma)^{\alpha_2 - 1}}{\beta_2^{\alpha_2}} e^{-(x-\gamma)^{\alpha_2} / \beta_2^{\alpha_2}}}{1 - (1 - e^{-(x-\gamma)^{\alpha_2} / \beta_2^{\alpha_2}})}$$

$$= \frac{\dfrac{\alpha_2 (x-\gamma)^{\alpha_2 - 1}}{\beta_2^{\alpha_2}} e^{-(x-\gamma)^{\alpha_2} / \beta_2^{\alpha_2}}}{e^{-(x-\gamma)^{\alpha_2} / \beta_2^{\alpha_2}}}$$

$$= \frac{\alpha_2 (x-\gamma)^{\alpha_2 - 1}}{\beta_2^{\alpha_2}} \qquad (2.19)$$

Selanjutnya, variansi distribusi Weibull tiga parameter diperoleh dari persamaan (2.2), yaitu:

$$V(X) = E(X^2) - (E(X))^2$$

berdasarkan persamaan (2.2) maka, terlebih dahulu akan ditunjukkan rata-rata atau $E(X)$, sebagai berikut:

$$E(X) = \int_{-\infty}^{\infty} x \, . \, f(x) \, dx$$

$$= \int_{0}^{\infty} x . \frac{\alpha_2 (x-\gamma)^{\alpha_2 - 1}}{\beta_2^{\alpha_2}} e^{-(x-\gamma)^{\alpha_2} / \beta_2^{\alpha_2}} \, dx$$

misalkan:

$$\theta = \beta^{\alpha_2}$$

$$z = (x-\gamma)^{\alpha_2}$$

$$z^{1/\alpha_2} = (x-\gamma)$$

$$z^{1/\alpha_2} + \gamma = x$$

$$dx = 1/\alpha_2 \, z^{1/\alpha_2 - 1} \, dz$$

maka:

$$E(X) = \int_{0}^{\infty} (z^{1/\alpha_2} + \gamma) \frac{\alpha_2 \, z.z^{-1/\alpha_2}}{\theta} e^{-z/\theta} . \, 1/\alpha_2 \, z^{1/\alpha_2 - 1} \, dz$$

$$= \int_{0}^{\infty} (z^{1/\alpha_2} + \gamma) \frac{\alpha_2 \, z.z^{-1/\alpha_2}}{\theta} e^{-z/\theta} . \, 1/\alpha_2 \, z^{1/\alpha_2 - 1} \, dz$$

$$E(X)=\int_0^\infty (z^{1/\alpha_2}+\gamma)\frac{1}{\theta}e^{-z/\theta}\,dz$$
$$=\int_0^\infty (z^{1/\alpha_2}\frac{1}{\theta}e^{-z/\theta}+\gamma\frac{1}{\theta}e^{-z/\theta})dz$$
$$=\frac{1}{\theta}\int_0^\infty z^{1/\alpha_2}.e^{-z/\theta}\,dz+\frac{\gamma}{\theta}\int_0^\infty e^{-z/\theta}\,dz$$
$$=\frac{1}{\theta}\int_0^\infty z^{1/\alpha_2}.e^{-z/\theta}\,\frac{\theta^{1/\alpha_2+1}}{\theta^{1/\alpha_2+1}}\frac{\Gamma(1/\alpha_2+1)}{\Gamma(1/\alpha_2+1)}dz+\frac{\gamma\theta}{\theta}e^{-z/\theta}\Big|_0^\infty$$
$$=\frac{\theta^{1/\alpha_2+1}\Gamma(1/\alpha_2+1)}{\theta}\int_0^\infty \frac{z^{1/\alpha_2}.e^{-z/\theta}}{\theta^{1/\alpha_2+1}\Gamma(1/\alpha_2+1)}dz-\frac{\gamma\theta}{\theta}e^{-z/\theta}\Big|_0^\infty$$

dengan:

$$\int_0^\infty \frac{z^{1/\alpha_2}.e^{-z/\theta}}{\theta^{1/\alpha_2+1}\Gamma(1/\alpha_2+1)}dz=1$$

maka:

$$E(X)=\frac{\theta^{1/\alpha_2+1}\Gamma(1/\alpha_2+1)}{\theta}.1-((\gamma.0)-(\gamma.1))$$
$$=\frac{\theta^{1/\alpha_2+1}\Gamma(1/\alpha_2+1)}{\theta}+\gamma$$

dengan $\theta=\beta_2^{\alpha_2}$, maka:

$$E(X)=\beta_2\Gamma(1/\alpha_2+1)+\gamma \tag{2.20}$$

selanjutnya akan ditunjukkan seperti dibawah ini:

$$E(X^2)=\int_{-\infty}^\infty x^2.f(x)\,dx$$
$$=\int_0^\infty x^2.\frac{\alpha_2(x-\gamma)^{\alpha_2-1}}{\beta_2^{\alpha_2}}e^{-(x-\gamma)^{\alpha_2}/\beta_2^{\alpha_2}}\,dx$$

misalkan:

$$\theta = \beta^{\alpha_2}$$
$$z = (x-\gamma)^{\alpha_2}$$
$$z^{1/\alpha_2} = (x-\gamma)$$
$$z^{1/\alpha_2} + \gamma = x$$
$$dx = \frac{1}{\alpha_2}\, z^{1/\alpha_2 - 1}\, dz$$

sehingga:

$$E(X^2) = \int_0^\infty (z^{1/\alpha_2} + \gamma)^2 \,.\frac{\alpha_2\; z.z^{-1/\alpha}}{\theta} e^{-z/\theta}.\, \frac{1}{\alpha_2}\, z^{1/\alpha_2 - 1}\, dz$$
$$= \int_0^\infty (z^{1/\alpha_2} + \gamma)^2 \,.\frac{e^{-z/\theta}}{\theta}.dz$$
$$= \int_0^\infty (z^{2/\alpha_2} \frac{e^{-z/\theta}}{\theta} dz + 2\gamma \int_0^\infty z^{1/\alpha_2} \frac{e^{-z/\theta}}{\theta} dz + \gamma^2 \int_0^\infty \frac{e^{-z/\theta}}{\theta}.dz$$

selanjutnya bentuk ke dalam fungsi densitas peluang Gamma, maka:

$$E(X^2) = \int_0^\infty (z^{2/\alpha} \frac{e^{-z/\theta}}{\theta} \frac{\theta^{2/\alpha+1}\Gamma(2/\alpha+1)}{\theta^{2/\alpha+1}\Gamma(2/\alpha+1)} dz + 2\gamma \int_0^\infty z^{1/\alpha} \frac{e^{-z/\theta}}{\theta} \frac{\theta^{1/\alpha+1}\Gamma(1/\alpha+1)}{\theta^{1/\alpha+1}\Gamma(1/\alpha+1)} dz$$
$$-\frac{\gamma^2\theta}{\theta} e^{-z/\theta}\Big|_0^\infty$$
$$= \frac{\theta^{2/\alpha+1}\Gamma(2/\alpha+1)}{\theta} \int_0^\infty z^{2/\alpha} \frac{e^{-z/\theta}}{\theta^{2/\alpha+1}\Gamma(2/\alpha+1)} dz +$$
$$\frac{2\gamma}{\theta}\theta^{1/\alpha_2+1}\Gamma(1/\alpha_2+1)\int_0^\infty z^{1/\alpha_2} \frac{e^{-z/\theta}}{\theta^{1/\alpha_2+1}\Gamma(1/\alpha_2+1)} dz - (\gamma^2.0 - (\gamma^2.1))$$
$$= \frac{\theta^{2/\alpha_2+1}\Gamma(2/\alpha_2+1)}{\theta}.1 + \frac{2\gamma}{\theta}\theta^{1/\alpha_2+1}\Gamma(1/\alpha_2+1).1 + \gamma^2$$
$$= \frac{\theta^{2/\alpha_2+1}\Gamma(2/\alpha_2+1)}{\theta} + \frac{2\gamma}{\theta}\theta^{1/\alpha_2+1}\Gamma(1/\alpha_2+1) + \gamma^2$$

dengan $\theta = \beta_2^{\alpha_2}$, maka:

$$E(X^2) = \frac{\theta^{2/\alpha_2+1}\Gamma(2/\alpha_2+1)}{\theta} + \frac{2\gamma}{\theta}\theta^{1/\alpha_2+1}\Gamma(1/\alpha_2+1) + \gamma^2$$
$$= \beta_2^2\,\Gamma(2/\alpha_2+1) + 2\gamma\beta_2\Gamma(1/\alpha_2+1) + \gamma^2 \qquad (2.21)$$

Berdasarkan persamaan (2.20) dan (2.21) dengan menggunakan persamaan (2.2), maka diperoleh variasi distribusi Weibull tiga parameter yaitu:

$$V(X) = E(X^2) - (E(X))^2$$

$$= \beta_2^2\,\Gamma(2/\alpha_2 + 1) + 2\gamma\beta_2\Gamma(1/\alpha_2 + 1) + \gamma^2 - (\beta_2\,\Gamma(1/\alpha_2 + 1) + \gamma))^2$$

$$= \beta_2^2\,\Gamma(2/\alpha_2 + 1) + 2\gamma\beta_2\Gamma(1/\alpha_2 + 1) + \gamma^2$$

$$-(\beta_2^2(\Gamma(1/\alpha_2 + 1))^2 + 2\gamma\beta_2\,\Gamma(1/\alpha_2 + 1) + \gamma^2$$

$$= \beta_2^2(\Gamma(2/\alpha_2 + 1) - (\Gamma(1/\alpha_2 + 1))^2) \qquad \blacksquare \qquad (2.22)$$

2.3 Parameter Campuran (w)

Parameter campuran w, sebagai proporsi pada sebuah kombinasi distribusi, dengan nilai $0 < w_i < 1$ dan $\sum w_i = 1,\ i = 1, 2, 3, ..., n$ Kabir (1968) secara umum memberikan asumsi metode campuran dari :

$$f(x) = \sum_{i=1}^{n} w_i \gamma_i \qquad (2.23)$$

yang mana $w_i \geq 0, \sum w_i = 1,$ parameter distribusi Weibull selanjutnya ditetapkan persamaan :

$$\sum_{i=1}^{n} A_i \lambda_i (\mu_i)^j$$

yang mana A_i konstanta tidak nol, dengan fungsi μ_i dan λ_i adalah fungsi monoton.

2.4 Distribusi Weibull Campuran

Distribusi campuran adalah distribusi yang dibentuk dari kombinasi dua atau lebih komponen distribusi. Model Weibull campuran dapat ditunjukkan dengan:

$$F(x) = wF_1(x) + (1-w)F_2(x) \qquad (2.24)$$

dengan $F_1(x),\ F_2(x)$ diberikan oleh persamaan (2.14) dan (2.17), dimana model yang akan terbentuk merupakan karakteristrik parameter lima. Fungsi peluang densitas dan fungsi Hazard diberikan oleh:

$$f(x) = wf_1(x) + (1-w)f_2(x) \qquad (2.25)$$

dan,

$$h(t) = \frac{wS_1(t)}{wS_1(t)+(1-w)S_2(t)}h_1(t) + \frac{(1-w)S_2(t)}{wS_1(t)+(1-w)S_2(t)}h_2(t) \qquad (2.26)$$

dengan,

$$\sum w_i = 1$$

dengan w_i adalah parameter campuran dengan peluang pada komponen distribusi campuran. Fungsi $f_1(x)$ dan $f_2(x)$ adalah fungsi densitas peluang pada komponen distribusi Weibull dua parameter dan tiga parameter.

2.5 Estimasi Maximum *Likelihood*

Jiang dan Kececioglu (1992) menunjukkan algoritma untuk mengestimasi parameter pada Weibull campuran menggunakan maksimum *likelihood* dalam menentukan nilai awal suatu parameter.

Metode maksimum *likelihood* adalah metode yang paling baik digunakan untuk memperoleh sebuah estimator beberapa parameter yang tidak diketahui. Misalkan $y_1, y_2, ..., y_n$ adalah sampel variabel random kontinu $y_1, y_2, .., y_n$ dan *likelihood* L akan ditunjukkan dengan $L \; y_1, y_2, ..., y_n$ fungsi nyata pada $y_1, y_2, .., y_n$. Fungsi *likelihood* ditunjukkan dengan :

$$L = f(x_1).f(x_2)...f(x_n) \text{ atau } L = \prod_{i=1}^{n} f(xi)$$

Dengan memaksimumkan fungsi selanjutnya akan digunakan pendekatan numerik yaitu Newton Raphson. Metode ini menggunakan iterasi numerik yang digunakan untuk menyelesaikan persamaan nonlinier dalam menentukan akar-akar persamaan. Misalkan nilai $x_1, x_2, ...x_n$ yang ditunjukkan sebagai berikut:

$$f_1(x_1 ....x_n) = 0$$

$$f_2(x_1 ....x_n) = 0$$

$$\vdots$$

$$f_n(x_1 ....x_n) = 0$$

misalkan a_{ij} turunan parsial dari fungsi f_i terhadap x_j sehingga $a_{ij} = \partial f_i / \partial x_j$. Dapat ditunjukkan dengan matrik jacobian seperti dibawah ini:

$$J = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & \ddots & \vdots \\ a_{n1} & \cdots & \cdots & a_{nn} \end{bmatrix}$$

misalkan invers matrik jacobian dinotasikan dengan J^{-1} yang ditunjukkan sebagai berikut:

$$J^{-1} = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & & \ddots & \vdots \\ b_{n1} & \cdots & \cdots & b_{nn} \end{bmatrix}$$

selanjutnya misalkan $x_1^k, x_2^k, ..., x_n^k$ akan dihampiri dengan k iterasi, misalkan

$$f_1^k(x_1 ....x_n) = 0$$

$$f_2^k(x_1 ....x_n) = 0$$

$$\vdots$$

$$f_n^k(x_1 ....x_n) = 0$$

dan misalkan b_{ij}^k dengan ij elemen pada J^{-1} yang memiliki $x_1^k, x_2^k, ..., x_n^k$, selanjutnya aproksimasi atau perkiraan diberikan oleh:

$$x_1^{k+1} = x_1^k - (b_{11}^k f_1^k + b_{12}^k f_2^k + ... + b_{1n}^{k+1} f_n^k)$$

$$x_2^{k+1} = x_2^k - (b_{21}^k f_1^k + b_{22}^k f_2^k + ... + b_{2n}^{k+1} f_n^k)$$

$$\vdots$$

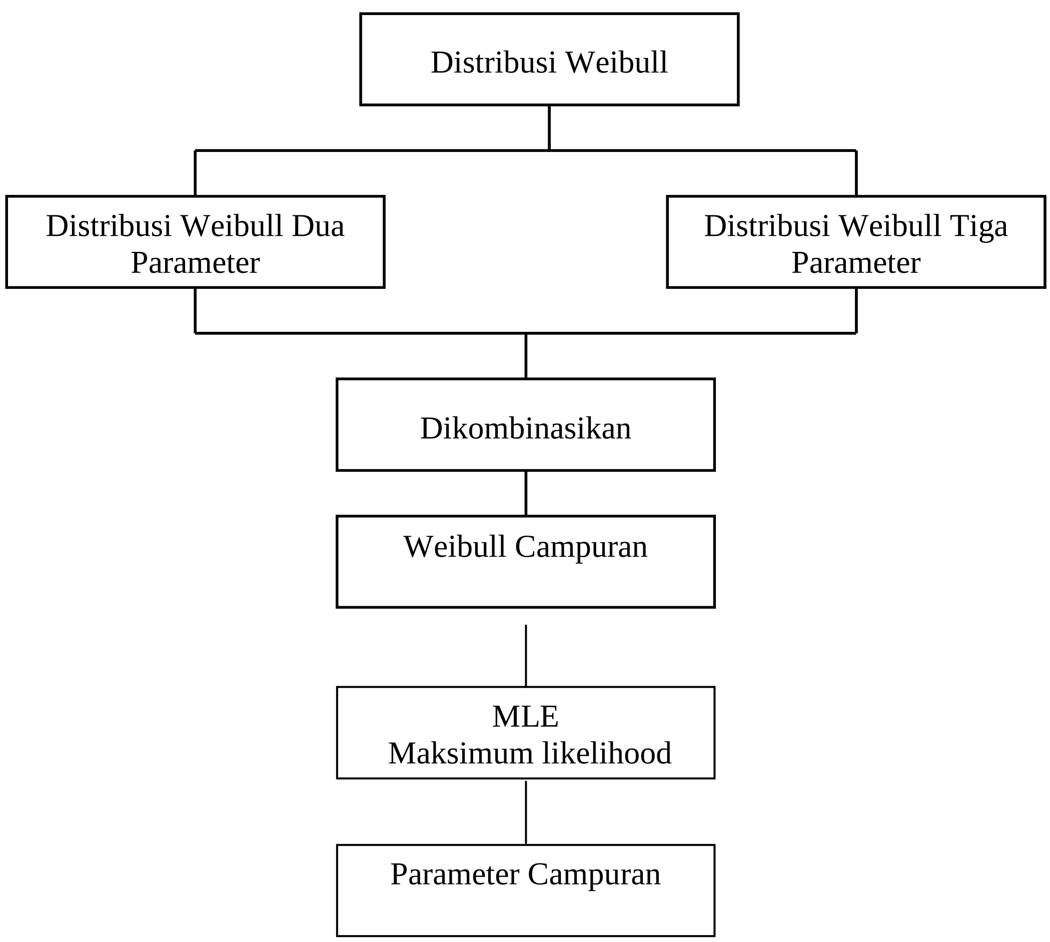
$$x_n^{k+1} = x_n^k - (b_{n1}^k f_1^k + b_{n2}^k f_2^k + ... + b_{nn}^{k+1} f_n^k)$$

BAB III

METODOLOGI PENELITIAN

Penyusunan tugas akhir ini penulis mengambil objek penelitian mengenai distribusi kontinu yaitu distribusi Weibull. Pada tahap pertama penulis mengambil distribusi weibull dua parameter dan tiga parameter yang selanjutnya akan dicari nilai rata-rata, variasi, fungsi kumulatif, fungsi survival dan fungsi hazard.

Selanjutnya penulis menggabungkan kedua distribusi dengan menggunakan parameter campuran, dimana parameter campuran yang digunakan adalah W dan $(1-W)$. Selanjutnya dengan menggunakan estimasi maksimum *likelihood* akan diperoleh nilai awal dari suatu parameter Weibull campuran atau yang lebih dikenal Weibull gabungan. Berikut adalah flowchart metodologi dari tugas akhir ini, yaitu:



Gambar 3.1 *Flowchart* metodologi penelitian

BAB IV

KOMBINASI DUA DISTRIBUSI WEIBULL DENGAN MENGGUNAKAN PARAMETER CAMPURAN

Bab ini akan membahas tentang kombinasi dua distribusi Weibull dengan menggunakan parameter campuran yang sering disebut juga distribusi Weibull campuran. Pembahasan ini dimulai dengan mengkombinasikan dua buah distribusi Weibull dengan menggunakan parameter campuran yang selanjutnya akan ditentukan nilai awal dari distribusi Weibull campuran.

4.1 Distribusi Weibull Campuran

Distribusi Weibull campuran dapat dibentuk berdasarkan persamaan (2.25) yang mana f_1 dan f_2 merupakan fungsi densitas peluang dari distribusi Weibul parameter dua dan tiga, yang dapat ditunjukkan sebagai berikut:

$$f(x) = wf_1(x) + (1-w)\,f_2(x)$$

maka:

$$f(x) \quad = w\left(\alpha_1 \frac{x^{\alpha_1-1}}{\beta_1^{\alpha_1}}\right).e^{-\left(x/\beta_1\right)^{\alpha_1}} + (1-w)\left(\frac{\alpha_2\,(x-\gamma)^{\alpha_2-1}}{\beta_2^{\alpha_2}}\right).e^{-\left(x/\beta_2\right)^{\alpha_2}} \tag{4.1}$$

Suatu distribusi dikatakan kontinu apabila memenuhi syarat $\int_{-\infty}^{\infty} f(x)\,dx = 1$. Selanjutnya akan ditunjukkan bahwa distribusi Weibull pada persamaan (4.1) memenuhi syarat distribusi peluang kontinu, sebagai berikut:

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

yang mana,

$$f(x) \quad = w\left(\alpha_1 \frac{x^{\alpha_1-1}}{\beta_1^{\alpha_1}}\right).e^{-\left(x/\beta_1\right)^{\alpha_1}} + (1-w)\left(\frac{\alpha_2\,(x-\gamma)^{\alpha_2-1}}{\beta_2^{\alpha_2}}\right).e^{-\left(x/\beta_2\right)^{\alpha_2}}$$

maka:

$$\int_{-\infty}^{\infty} f(x)\,dx = 1 \quad \int_{-\infty}^{\infty}\left[w\left(\alpha_1 \frac{x^{\alpha_1-1}}{\beta_1^{\alpha_1}}\right).e^{-\left(x/\beta_1\right)^{\alpha_1}} + (1-w)\left(\frac{\alpha_2\,(x-\gamma)^{\alpha_2-1}}{\beta_2^{\alpha_2}}\right).e^{-\left(x/\beta_2\right)^{\alpha_2}}\right] = 1$$

$$\int_0^x w\left(\alpha_1 \frac{x^{\alpha_1-1}}{\beta_1^{\alpha_1}}\right).e^{-\left(x/\beta_1\right)^{\alpha_1}}\,dx+\int_\gamma^x (1-w)\left(\frac{\alpha_2(x-\gamma)^{\alpha_2-1}}{\beta_2^{\alpha_2}}\right).e^{-\left(x/\beta_2\right)^{\alpha_2}}\,dx=1$$

dengan,

$$\int_0^x \left(\alpha_1 \frac{x^{\alpha_1-1}}{\beta_1^{\alpha_1}}\right).e^{-\left(x/\beta_1\right)^{\alpha_1}}\,dx=1$$

dan

$$\int_\gamma^x (1-w)\left(\frac{\alpha_2(x-\gamma)^{\alpha_2-1}}{\beta_2^{\alpha_2}}\right).e^{-\left(x/\beta_2\right)^{\alpha_2}}\,dx=1$$

maka:

$$w\int_0^x \left(\alpha_1 \frac{x^{\alpha_1-1}}{\beta_1^{\alpha_1}}\right).e^{-\left(x/\beta_1\right)^{\alpha_1}}\,dx+(1-w)\int_\gamma^x \left(\frac{\alpha_2(x-\gamma)^{\alpha_2-1}}{\beta_2^{\alpha_2}}\right).e^{-\left(x/\beta_2\right)^{\alpha_2}}\,dx=1$$

$$w.1+(1-w)=1$$

$$1=1$$

Berdasarkan persamaan (2.1) dan (2.2) akan ditunjukkan rata-rata dan variansi dari distribusi Weibull campuran, yang akan ditunjukkan sebagai berikut:

$$E(X)=\int_{-\infty}^{\infty} x.\,f(x)\,dx$$

yang mana diketahui bahwa:

$$f(x) \quad = w\left(\alpha_1 \frac{x^{\alpha_1-1}}{\beta_1^{\alpha_1}}\right).e^{-\left(x/\beta_1\right)^{\alpha_1}} + (1-w)\left(\frac{\alpha_2(x-\gamma)^{\alpha_2-1}}{\beta_2^{\alpha_2}}\right).e^{-\left(x/\beta_2\right)^{\alpha_2}}$$

sehingga:

$$E(X)=\int_{-\infty}^{\infty} x.\,f(x)\,dx$$

$$=\int_{-\infty}^{\infty} x.\left(w\left(\alpha_1 \frac{x^{\alpha_1-1}}{\beta_1^{\alpha_1}}\right).e^{-\left(x/\beta_1\right)^{\alpha_1}} + (1-w)\left(\frac{\alpha_2(x-\gamma)^{\alpha_2-1}}{\beta_2^{\alpha_2}}\right).e^{-\left(x/\beta_2\right)^{\alpha_2}}\right)dx$$

sehingga dapat dibentuk menjadi:

$$E(X)=\int_{-\infty}^{\infty} x.w\left(\alpha_1 \frac{x^{\alpha_1-1}}{\beta_1^{\alpha_1}}\right).e^{-\left(x/\beta_1\right)^{\alpha_1}}\,dx+\int_{-\infty}^{\infty} x.(1-w)\left(\frac{\alpha_2(x-\gamma)^{\alpha_2-1}}{\beta_2^{\alpha_2}}\right).e^{-\left(x/\beta_2\right)^{\alpha_2}}\,dx$$

berdasarkan persamaan (2.11) dan (2.20) maka diperoleh:

$$E(X)=\beta_1\,\Gamma(1/\alpha_1+1)+\beta_2\,\Gamma(1/\alpha_2+1)+\gamma \tag{4.2}$$

Selanjutnya untuk mencari variansi distribusi Weibull campuran berdasarkan persamaan (2.2) maka terlebih dahulu akan dicari nilai rata-rata kuadrat, yang ditunjukkan sebagai berikut:

$$E(X^2) = \int_{-\infty}^{\infty} x^2 \ f(x)\,dx$$

$$= \int_{-\infty}^{\infty} x^2 \left(w\left(\alpha_1 \frac{x^{\alpha_1 - 1}}{\beta_1^{\alpha_1}}\right).e^{-\left(x/\beta_1\right)^{\alpha_1}} + (1-w)\left(\frac{\alpha_2 (x-\gamma)^{\alpha_2 - 1}}{\beta_2^{\alpha_2}}\right).e^{-\left(x/\beta_2\right)^{\alpha_2}} \right) dx$$

$$= \int_{0}^{\infty} x^2 w\left(\alpha_1 \frac{x^{\alpha_1 - 1}}{\beta_1^{\alpha_1}}\right).e^{-\left(x/\beta_1\right)^{\alpha_1}}\,dx + \int_{0}^{\infty} x^2 (1-w)\left(\frac{\alpha_2 (x-\gamma)^{\alpha_2 - 1}}{\beta_2^{\alpha_2}}\right).e^{-\left(x/\beta_2\right)^{\alpha_2}}\,dx$$

berdasarkan persamaan (2.12) dan (2.21) sehingga diperoleh:

$$E(x^2) = \beta_1^{\,2}\,\Gamma(2/\alpha_1 + 1) + \beta_2^{\,2}\,\Gamma(2/\alpha_2 + 1) + 2\,\gamma\beta_2\Gamma(1/\alpha_2 + 1) + \gamma^2 \qquad (4.3)$$

maka berdasarkan persamaan (4.2) dan (4.3) maka variansi distribusi Weibull campuran dapat ditunjukkan sebagai berikut:

$$V(X) = E(X^2) - (E(X))^2$$

$$= \left(\beta_1^{\,2}\,\Gamma(2/\alpha_1 + 1) + \beta_2^{\,2}\,\Gamma(2/\alpha_2 + 1) + 2\,\gamma\beta_2\Gamma(1/\alpha_2 + 1) + \gamma^2\right) -$$

$$\left(\beta_1\,\Gamma(1/\alpha_1 + 1) + \beta_2\,\Gamma(1/\alpha_2 + 1) + \gamma\right)^2 \qquad (4.4)$$

Selanjutnya akan ditunjukkan fungsi kumulatif dan fungsi Hazard dari distribusi Weibull campuran berdasarkan persamaan (2.4) dan (2.6), sebagai berikut:

$$F(x) = wF_1(x) + (1-w)F_2(x)$$

$$= w(1 - e^{-\left(x/\beta\right)^{\alpha}}) + (1-w)(1 - e^{-(x-\gamma)^{\alpha}/\beta^{\alpha}})$$

$$= -we^{-\left(x/\beta\right)^{\alpha}} - e^{-(x-\gamma)^{\alpha}/\beta^{\alpha}} + we^{-(x-\gamma)^{\alpha}/\beta^{\alpha}} + 1 \qquad (4.4)$$

yang mana $F_1(x)$ dan $F_2(x)$ merupakan fungsi kumulatif dari persamaan (2.14) dan (2.17) distribusi Weibull dua parameter dan tiga parameter.

Berdasarkan persamaan (2.26) maka fungsi Hazard dari distribusi Weibull campuran dapat ditunjukkan sebagai berikut:

$$h(t) = \frac{wS_1(t)}{wS_1(t) + (1-w)S_2(t)} h_1(t) + \frac{(1-w)S_2(t)}{wS_1(t) + (1-w)S_2(t)} h_2(t)$$

$$= \frac{we^{-\left(x/\beta\right)^{\alpha}}}{we^{-\left(x/\beta\right)^{\alpha}} + (1-w)e^{-\left(x-\gamma/\beta\right)^{\alpha}}} \frac{\alpha}{\beta^{\alpha}} x^{\alpha - 1}$$

$$+\frac{(1-w)e^{-\left(\frac{x-\gamma}{\beta}\right)^{\alpha}}}{we^{-\left(\frac{x}{\beta}\right)^{\alpha}}+(1-w)e^{-\left(\frac{x-\gamma}{\beta}\right)^{\alpha}}}\frac{\alpha}{\beta^{\alpha}}(x-\gamma)^{\alpha-1}$$

$$h(t)=\frac{we^{-\left(\frac{x}{\beta}\right)^{\alpha}}\frac{\alpha}{\beta^{\alpha}}x^{\alpha-1}+\left[e^{-\left(\frac{x-\gamma}{\beta}\right)^{\alpha}}-we^{-\left(\frac{x-\gamma}{\beta}\right)^{\alpha}}\right]\frac{\alpha}{\beta^{\alpha}}(x-\gamma)^{\alpha-1}}{we^{-\left(\frac{x}{\beta}\right)^{\alpha}}+(1-w)e^{-\left(\frac{x-\gamma}{\beta}\right)^{\alpha}}} \tag{4.4}$$

4.2 Nilai Awal Parameter Campuran

Nilai awal dari suatu parameter campuran dapat dicari dengan menggunakan metode maksimum *likelihood*, yakni memaksimumkan perkalian fungsi densitas Weibull campuran. Dapat ditunjukkan sebagai berikut:

$$L = f(x_1).f(x_2)...f(x_n)$$

sehingga *likelihood*:

$$L=\left(w\left(\frac{\alpha_1 x_1^{\alpha_1-1}e^{-(x_1/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right)+(1-w)\left(\frac{\alpha_2(x_1-\gamma)^{\alpha 2-1}e^{-(x_1-\gamma/\beta_2)^{\alpha_2}}}{\beta_2^{\alpha_2}}\right)\right)....$$

$$\left(w\left(\frac{\alpha_1 x_n^{\alpha_1-1}e^{-(x_1/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right)+(1-w)\left(\frac{\alpha_2(x_n-\gamma)^{\alpha 2-1}e^{-(x_1-\gamma/\beta_2)^{\alpha_2}}}{\beta_2^{\alpha_2}}\right)\right)$$

maka log *likehood* ditunjukkan dengan:

$$\ln L=\ln\left(w\left(\frac{\alpha_1 x_1^{\alpha_1-1}e^{-(x_1/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right)+(1-w)\left(\frac{\alpha_2(x_1-\gamma)^{\alpha 2-1}e^{-(x_1-\gamma/\beta_2)^{\alpha_2}}}{\beta_2^{\alpha_2}}\right)\right)....+$$

$$\ln\left(w\left(\frac{\alpha_1 x_n^{\alpha_1-1}e^{-(x_1/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right)+(1-w)\left(\frac{\alpha_2(x_n-\gamma)^{\alpha 2-1}e^{-(x_1-\gamma/\beta_2)^{\alpha_2}}}{\beta_2^{\alpha_2}}\right)\right)$$

$$=\sum_{i=1}^{n}\ln\left(w\left(\frac{\alpha_1 x_n^{\alpha_1-1}e^{-(x_1/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right)+(1-w)\left(\frac{\alpha_2(x_n-\gamma)^{\alpha 2-1}e^{-(x_1-\gamma/\beta_2)^{\alpha_2}}}{\beta_2^{\alpha_2}}\right)\right)$$

selanjutnya dengan memaksimumkan fungsi *likelihood* maka:

$$1.\ \frac{d\ln L}{\alpha_1}=\sum_{i=1}^{n}\left(\frac{1}{\left(w\left(\frac{\alpha_1 x_i^{\alpha_1-1}e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right)+(1-w)\left(\frac{\alpha_2(x_i-\gamma)^{\alpha 2-1}e^{-(x_i-\gamma/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right)\right)}.\frac{df_1}{d\alpha_1}\right)$$

$$=\sum_{i=1}^{n}\left(\frac{\frac{wx_i^{\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}+w\alpha_1 x_i^{\alpha_1-1}\ln x_i\,.\,e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}-w\alpha_1 x_i^{\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}\ln\left(\frac{x_i}{\beta_1}\right)}{\beta_1^{\alpha_1}}}{\left(w\left(\frac{\alpha_1 x_i^{\alpha_1-1}e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right)+(1-w)\left(\frac{\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-(x_i-\gamma/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right)\right)}-\frac{\frac{wx_i^{\alpha_1-1}\alpha_1 e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}\ln\beta_1}{\beta_1^{\alpha_1}}}{\left(w\left(\frac{\alpha_1 x_i^{\alpha_1-1}e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right)+(1-w)\left(\frac{\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-(x_i-\gamma/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right)\right)}\right)$$

$$2.\ \frac{d\ln L}{d\beta_1}=\sum_{i=1}^{n}\left(\frac{1}{\left(w\left(\frac{\alpha_1 x_i^{\alpha_1-1}e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right)+(1-w)\left(\frac{\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-(x_i-\gamma/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right)\right)}.\frac{df_1}{d\beta_1}\right)$$

$$=\sum_{i=1}^{n}\left(\frac{\left(\frac{w\alpha_1^2 x_i^{\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}-w\alpha_1^2 x_i^{\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}}{\beta_1\beta_1^{\alpha_1}}\right)}{\left(w\left(\frac{\alpha_1 x_i^{\alpha_1-1}e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right)+(1-w)\left(\frac{\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-(x_i-\gamma/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right)\right)}\right)$$

$$3.\ \frac{d\ln L}{\alpha_2}=\sum_{i=1}^{n}\left(\frac{1}{\left(w\left(\frac{\alpha_1 x_i^{\alpha_1-1}e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right)+(1-w)\left(\frac{\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-(x_i-\gamma/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right)\right)}\cdot\frac{df_2}{d\alpha_2}\right)$$

$$=\sum_{i=1}^{n}\left(\frac{\left(\frac{(1-w)(x_i-\gamma)^{\alpha_2-1}e^{-(x_i-\gamma/\beta_2)^{\alpha 2}}+(1-w)\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-(x_i-\gamma/\beta_2)^{\alpha 2}}\ln(x_i-\gamma)}{\beta_2^{\alpha_2}}\right.}{\left(w\left(\frac{\alpha_1 x_i^{\alpha_1-1}e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right)+(1-w)\left(\frac{\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-(x_i-\gamma/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right)\right)}\right.$$

$$-\frac{\frac{(1-w)\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}\ln\left(\frac{x_i-\gamma}{\beta_2}\right)}{\beta_2^{\alpha_2}}}{\left(w\left(\frac{\alpha_1 x_i^{\alpha_1-1}e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right)+(1-w)\left(\frac{\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-(x_i-\gamma/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right)\right)}$$

$$\left.-\frac{\left.\frac{(1-w)\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\ln\beta_2}{\beta_2^{\alpha_2}}\right)}{\left(w\left(\frac{\alpha_1 x_i^{\alpha_1-1}e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right)+(1-w)\left(\frac{\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-(x_i-\gamma/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right)\right)}\right)$$

4. $$\frac{d\ln L}{\beta_2}=\sum_{i=1}^{n}\left(\frac{1}{\left(w\left(\frac{\alpha_1 x_i^{\alpha_1-1}e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right)+(1-w)\left(\frac{\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-(x_i-\gamma/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right)\right)}\cdot\frac{df_2}{d\beta_2}\right)$$

$$=\sum_{i=1}^{n}\left(\frac{\frac{(1-w)\alpha_2^2(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha 2}}\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}-\left((1-w)\alpha_2^2(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha 2}}\right)}{\beta_2\beta_2^{\alpha_2}}}{\left(w\left(\frac{\alpha_1 x_i^{\alpha_1-1}e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right)+(1-w)\left(\frac{\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-(x_i-\gamma/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right)\right)}\right)$$

5. $$\frac{d\ln L}{\gamma}=\sum_{i=1}^{n}\left(\frac{1}{\left(w\left(\frac{\alpha_1 x_i^{\alpha_1-1}e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right)+(1-w)\left(\frac{\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-(x_i-\gamma/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right)\right)}\cdot\frac{df_2}{d\gamma}\right)$$

$$=\sum_{i=1}^{n}\left(\frac{-\frac{(1-w)\alpha_2(x_i-\gamma)^{\alpha_2-1}(\alpha_2-1)e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha 2}}}{(x_i-\gamma)\beta_2^{\alpha_2}}}{\left(w\left(\frac{\alpha_1 x_i^{\alpha_1-1}e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right)+(1-w)\left(\frac{\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-(x_i-\gamma/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right)\right)}\right.$$

$$\left.+\frac{\frac{(1-w)\alpha_2^2(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha 2}}\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}{(x_i-\gamma)\beta_2^{\alpha_2}}}{\left(w\left(\frac{\alpha_1 x_i^{\alpha_1-1}e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right)+(1-w)\left(\frac{\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-(x_i-\gamma/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right)\right)}\right)$$

6. $$\frac{d\ln L}{w}=\sum_{i=1}^{n}\left(\frac{1}{\left(w\left(\frac{\alpha_1 x_i^{\alpha_1-1}e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right)+(1-w)\left(\frac{\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-(x_i-\gamma/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right)\right)}.\frac{df}{dw}\right)$$

$$=\sum_{i=1}^{n}\left(\frac{\left(\frac{\alpha_1 x_i^{\alpha_1-1}e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right)-\left(\frac{\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-(x_i-\gamma/\beta_1)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right)}{\left(w\left(\frac{\alpha_1 x_i^{\alpha_1-1}e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right)+(1-w)\left(\frac{\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-(x_i-\gamma/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right)\right)}\right)$$

Selanjutnya akan ditentukan nilai awal dari distribusi Weibull campuran dengan menggunakan matrik jacobian. Matrik J adalah matrik jacobian dari distribusi Weibull campuran, yaitu:

$$J=\begin{bmatrix}\frac{d1}{d\alpha_1} & \frac{d1}{d\beta_1} & \frac{d1}{d\alpha_2} & \frac{d1}{d\beta_2} & \frac{d1}{d\gamma} & \frac{d1}{dw}\\ \frac{d2}{d\alpha_1} & \frac{d2}{d\beta_1} & \frac{d2}{d\alpha_2} & \frac{d2}{d\beta_2} & \frac{d2}{d\gamma} & \frac{d2}{dw}\\ \frac{d3}{d\alpha_1} & \frac{d3}{d\beta_1} & \frac{d3}{d\alpha_2} & \frac{d3}{d\beta_2} & \frac{d3}{d\gamma} & \frac{d3}{dw}\\ \frac{d4}{d\alpha_1} & \frac{d4}{d\beta_1} & \frac{d4}{d\alpha_2} & \frac{d4}{d\beta_2} & \frac{d4}{d\gamma} & \frac{d4}{dw}\\ \frac{d5}{d\alpha_1} & \frac{d5}{d\beta_1} & \frac{d5}{d\alpha_2} & \frac{d5}{d\beta_2} & \frac{d5}{d\gamma} & \frac{d5}{dw}\\ \frac{d6}{d\alpha_1} & \frac{d6}{d\beta_1} & \frac{d6}{d\alpha_2} & \frac{d6}{d\beta_2} & \frac{d6}{d\gamma} & \frac{d6}{dw}\end{bmatrix}$$

sehingga:

1. Akan ditunjukkan turunan dari persamaan (1) pada halaman IV-3 terhadap α_1, sebagai berikut:

$$\frac{d1}{d\alpha_1}=\sum_{i=1}^{n}\left(\frac{\frac{2wx_i^{\alpha_1-1}\ln x_i.e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}-\left(2wx_i^{\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}\ln\left(\frac{x_i}{\beta_1}\right)\right)}{\beta_1^{\alpha_1}}}{\left(w\left(\frac{\alpha_1 x_i^{\alpha_1-1}e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right)+(1-w)\left(\frac{\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-(x_i-\gamma/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right)\right)}-\right.$$

$$
\frac{\dfrac{2wx_i^{\alpha_1-1}\ln\beta_1.e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}+w\alpha_1 x_i^{\alpha_1-1}\ln(x_i)^2 e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}}{\beta_1^{\alpha_1}}}{\left(w\left(\dfrac{\alpha_1 x_i^{\alpha_1-1}e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right)+(1-w)\left(\dfrac{\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-(x_i-\gamma/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right)\right)}-
$$

$$
\frac{\dfrac{2w\alpha_1 x_i^{\alpha_1-1}\ln x_i.e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}\left(\dfrac{x_i}{\beta_1}\right)^{\alpha_1}\ln\left(\dfrac{x_i}{\beta_1}\right)^{\alpha_1}-2w\alpha_1 x_i^{\alpha_1-1}\ln x_i.e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}\ln\beta_1}{\beta_1^{\alpha_1}}}{\left(w\left(\dfrac{\alpha_1 x_i^{\alpha_1-1}e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right)+(1-w)\left(\dfrac{\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-(x_i-\gamma/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right)\right)}
$$

$$
\frac{+\dfrac{2w\alpha_1 x_i^{\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}\left(\left(\dfrac{x_i}{\beta_1}\right)^{\alpha_1}\right)^2\ln\left(\dfrac{x_i}{\beta_1}\right)^2-w\alpha_1 x_i^{\alpha_1-1}.e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}\left(\dfrac{x_i}{\beta_1}\right)^{\alpha_1}\ln\left(\dfrac{x_i}{\beta_1}\right)^2}{\beta_1^{\alpha_1}}}{\left(w\left(\dfrac{\alpha_1 x_i^{\alpha_1-1}e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right)+(1-w)\left(\dfrac{\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-(x_i-\gamma/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right)\right)}
$$

$$
\left.\frac{+\dfrac{2w\alpha_1 x_i^{\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}\left(\dfrac{x_i}{\beta_1}\right)^{\alpha_1}\ln\left(\dfrac{x_i}{\beta_1}\right)\ln\beta_1+w\alpha_1 x_i^{\alpha_1-1}.e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}\ln(\beta_1)^2}{\beta_1^{\alpha_1}}}{\left(w\left(\dfrac{\alpha_1 x_i^{\alpha_1-1}e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right)+(1-w)\left(\dfrac{\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-(x_i-\gamma/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right)\right)}\right)-
$$

$$
\left(\frac{\dfrac{wx_i^{\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}+w\alpha_1 x_i^{\alpha_1-1}\ln(x_i)e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}-w\alpha_1 x_i^{\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}\left(\dfrac{x_i}{\beta_1}\right)^{\alpha_1}\ln\left(\dfrac{x_i}{\beta_1}\right)^{\alpha_1}}{\beta_1^{\alpha_1}}}{\left(w\left(\dfrac{\alpha_1 x_i^{\alpha_1-1}e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right)+(1-w)\left(\dfrac{\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-(x_i-\gamma/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right)\right)^2}\right.
$$

$$
\left.\left.-\frac{\dfrac{w\alpha_1 x_i^{\alpha_1-1} e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}} \ln(\beta_1)}{\beta_1^{\alpha_1}}}{\left(w\left(\dfrac{\alpha_1 x_i^{\alpha_1-1} e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right)+(1-w)\left(\dfrac{\alpha_2 (x_i-\gamma)^{\alpha_2-1} e^{-(x_i-\gamma/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right)\right)^2}\right)\right)
$$

2. Akan ditunjukkan turunan dari persamaan (1) pada halaman IV-3 terhadap β_1 sebagai berikut:

$$
\frac{d1}{d\beta_1}=\sum_{i=1}^{n}\left(\frac{1}{\left(w\left(\dfrac{\alpha_1 x_i^{\alpha_1-1} e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right)+(1-w)\left(\dfrac{\alpha_2 (x_i-\gamma)^{\alpha_2-1} e^{-(x_i-\gamma/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right)\right)}\right.
$$

$$
\left(\frac{2wx_i^{\alpha_1-1}.e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}\left(\dfrac{x_i}{\beta_1}\right)^{\alpha_1}\alpha_1-2wx_i^{\alpha_1-1}.e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}\alpha_1}{\beta_1\beta_1^{\alpha_1}}+\right.
$$

$$
\frac{w\alpha_1^2 x_i^{\alpha_1-1}\ln(x_i).e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}\left(\dfrac{x_i}{\beta_1}\right)^{\alpha_1}\alpha_1-w\alpha_1^2 x_i^{\alpha_1-1}\ln(x_i).e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}}{\beta_1\beta_1^{\alpha_1}}-
$$

$$
\frac{2w\alpha_1^2 x_i^{\alpha_1-1}.e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}\left(\left(\dfrac{x_i}{\beta_1}\right)^{\alpha_1}\right)^2\ln\left(\dfrac{x_i}{\beta_1}\right)+2w\alpha_1^2 x_i^{\alpha_1-1}.e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}\left(\dfrac{x_i}{\beta_1}\right)^{\alpha_1}}{\beta_1\beta_1^{\alpha_1}}
$$

$$
\left.\left.\frac{\ln\left(\dfrac{x_i}{\beta_1}\right)-w\alpha_1^2 x_i^{\alpha_1-1}.e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}\left(\dfrac{x_i}{\beta_1}\right)^{\alpha_1}\ln(\beta_1)+w\alpha_1^2 x_i^{\alpha_1-1}.e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}\ln(\beta_1)}{\beta_1\beta_1^{\alpha_1}}\right)\right)-
$$

$$\frac{1}{\left(w\left(\frac{\alpha_1 x_i^{\alpha_1-1}e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right)+(1-w)\left(\frac{\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-(x_i-\gamma/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right)\right)^2}\left(\frac{wx_i^{\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}}{\beta_1^{\alpha_1}}+\right.$$

$$\frac{w\alpha_1 x_i^{\alpha_1-1}\ln(x_i)e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}-w\alpha_1 x_i^{\alpha_1-1}\ln(x_i)e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}\ln\left(\frac{x_i}{\beta_1}\right)-}{\beta_1^{\alpha_1}}$$

$$\left.\frac{w\alpha_1 x_i^{\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}\ln(\beta_1)}{\beta_1^{\alpha_1}}\right)\left(\frac{w\alpha_1^2 x_i^{\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}-w\alpha_1^2 x_i^{\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}}{\beta_1\beta_1^{\alpha_1}}\right)\Bigg)$$

3. Akan ditunjukkan turunan dari persamaan (1) pada halaman IV-3 terhadap α_2 sebagai berikut:

$$\frac{d1}{d\alpha_2}=\sum_{i=1}^{n}\left(\frac{1}{\left(w\left(\frac{\alpha_1 x_i^{\alpha_1-1}e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right)+(1-w)\left(\frac{\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-(x_i-\gamma/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right)\right)^2}\right.$$

$$\left(\frac{wx_i^{\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}}{\beta_1^{\alpha_1}}+\frac{w\alpha_1 x_i^{\alpha_1-1}\ln(x_i)e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}-w\alpha_1 x_i^{\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}{\beta_1^{\alpha_1}}\right.$$

$$\left.\frac{\ln\left(\frac{x_i}{\beta_1}\right)-w\alpha_1 x_i^{\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}\ln\beta_1}{\beta_1^{\alpha_1}}\right)\left(\frac{(1-w)(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha 2}}+(1-w)}{\beta_2^{\alpha_2}}\right.$$

$$\frac{\alpha_2 (x_i-\gamma)^{\alpha_2-1}\ln(x_i-\gamma)e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha 2}} -(1-w)\alpha_2 (x_i-\gamma)^{\alpha_2-1} e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha 2}} \left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}{\beta_2^{\alpha_2}}$$

$$\left.\left.\frac{\ln\left(\frac{x_i-\gamma}{\beta_2}\right)-(1-w)\alpha_2 (x_i-\gamma)^{\alpha_2-1} e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha 2}} \ln(\beta_2)}{\beta_2^{\alpha_2}}\right)\right)$$

4. Akan ditunjukkan turunan dari persamaan (1) pada halaman IV-3 terhadap β_2 sebagai berikut:

$$\frac{d1}{d\beta_2}=\sum_{i=1}^{n}\left(\frac{1}{\left(w\left(\frac{\alpha_1 x_i^{\alpha_1-1}e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right)+(1-w)\left(\frac{\alpha_2 (x_i-\gamma)^{\alpha_2-1}e^{-(x_i-\gamma/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right)\right)^2}\right.$$

$$\left(\frac{w x_i^{\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}}{\beta_1^{\alpha_1}}+\frac{w\alpha_1 x_i^{\alpha_1-1}\ln(x_i)e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}} - w\alpha_1 x_i^{\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right.$$

$$\left.\frac{\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}\ln\left(\frac{x_i}{\beta_1}\right)-w\alpha_1 x_i^{\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}\ln\beta_1}{\beta_1^{\alpha_1}}\right)\left(\frac{(1-w)\alpha_2^2 (x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha 2}}}{\beta_2\beta_2^{\alpha_2}}\right.$$

$$\left.\left.\frac{\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}-(1-w)\alpha_2^2 (x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha 2}}}{\beta_2\beta_2^{\alpha_2}}\right)\right)$$

5. Akan ditunjukkan turunan dari persamaan (1) pada halaman IV-3 terhadap γ sebagai berikut:

$$\frac{d1}{d\gamma}=\sum_{i=1}^{n}\left(\frac{1}{\left(w\left(\frac{\alpha_1 x_i^{\alpha_1-1}e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right)+(1-w)\left(\frac{\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-(x_i-\gamma/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right)\right)^2}\right.$$

$$\left(\frac{wx_i^{\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}}{\beta_1^{\alpha_1}}+\frac{w\alpha_1 x_i^{\alpha_1-1}\ln(x_i)e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}-w\alpha_1 x_i^{\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}{\beta_1^{\alpha_1}}\right.$$

$$\left.\frac{\ln\left(\frac{x_i}{\beta_1}\right)-w\alpha_1 x_i^{\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}\ln\beta_1}{\beta_1^{\alpha_1}}\right)\left(\frac{-(1-w)\alpha_2(x_i-\gamma)^{\alpha_2-1}(\alpha_2-1)e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha 2}}}{(x_i-\gamma)\beta_2^{\alpha_2}}+\right.$$

$$\left.\left.\frac{(1-w)\alpha_2^2(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha 2}}\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}{(x_i-\gamma)\beta_2^{\alpha_2}}\right)\right)$$

6. Akan ditunjukkan turunan dari persamaan (1) pada halaman IV-3 terhadap w sebagai berikut:

$$\frac{d1}{dw}=\sum_{i=1}^{n}\left(\frac{1}{\left(w\left(\frac{\alpha_1 x_i^{\alpha_1-1}e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right)+(1-w)\left(\frac{\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-(x_i-\gamma/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right)\right)}\right.\left(\frac{x_i^{\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}}{\beta_1^{\alpha_1}}+\right.$$

$$\left.\left.\frac{\alpha_1 x_i^{\alpha_1-1}\ln(x_i)e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}} - \alpha_1 x_i^{\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}\ln\left(\frac{x_i}{\beta_1}\right) - \alpha_1 x_i^{\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}\ln\beta_1}{\beta_1^{\alpha_1}}\right)\right)$$

$$\frac{-1}{\left(w\left(\frac{\alpha_1 x_i^{\alpha_1-1}e^{-\left(x_i/\beta_1\right)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right)+(1-w)\left(\frac{\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-\left(x_i-\gamma/\beta_2\right)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right)\right)^2}$$

$$\left(\frac{wx_i^{\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}} + w\alpha_1 x_i^{\alpha_1-1}\ln(x_i)e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}} - w\alpha_1 x_i^{\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}\ln\left(\frac{x_i}{\beta_1}\right)}{\beta_1^{\alpha_1}} - \right.$$

$$\left.\frac{w\alpha_1 x_i^{\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}\ln\beta_1}{\beta_1^{\alpha_1}}\right)\left(\frac{\alpha_1 x_i^{\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}}{\beta_1^{\alpha_1}} - \frac{\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}}{\beta_2^{\alpha_2}}\right)\Bigg)$$

7. Akan ditunjukkan turunan dari persamaan (2) pada halaman IV-4 terhadap α_1 sebagai berikut:

$$\frac{d2}{d\alpha_1} = \sum_{i=1}^{n}\left(\frac{1}{\left(w\left(\frac{\alpha_1 x_i^{\alpha_1-1}e^{-\left(x_i/\beta_1\right)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right)+(1-w)\left(\frac{\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-\left(x_i-\gamma/\beta_2\right)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right)\right)}\right.$$

$$\left(\frac{2wx_i^{\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}\alpha_1 - 2wx_i^{\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}\alpha_1}{\beta_1\beta_1^{\alpha_1}} + \right.$$

$$\frac{w\alpha_1^2 x_i^{\alpha_1-1}\ln(x_i).e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}\alpha_1 - w\alpha_1^2 x_i^{\alpha_1-1}\ln(x_i).e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}}{\beta_1\beta_1^{\alpha_1}} -$$

$$\frac{2w\alpha_1^2 x_i^{\alpha_1-1}.e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}\left(\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}\right)^2\ln\left(\frac{x_i}{\beta_1}\right) + 2w\alpha_1^2 x_i^{\alpha_1-1}.e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}\ln\left(\frac{x_i}{\beta_1}\right)}{\beta_1\beta_1^{\alpha_1}} +$$

$$\left.\left.\frac{w\alpha_1^2 x_i^{\alpha_1-1}.e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}\ln\beta_1 - w\alpha_1^2 x_i^{\alpha_1-1}.e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}\ln(\beta_1)}{\beta_1\beta_1^{\alpha_1}}\right)\right) -$$

$$\frac{1}{\left(w\left(\frac{\alpha_1 x_i^{\alpha_1-1}e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right) + (1-w)\left(\frac{\alpha_2 (x_i-\gamma)^{\alpha_2-1}e^{-((x_i-\gamma)/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right)\right)^2}\left(\frac{wx_i^{\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}}{\beta_1^{\alpha_1}} +\right.$$

$$\left.\frac{w\alpha_1 x_i^{\alpha_1-1}\ln(x_i)e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}} - w\alpha_1 x_i^{\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}\ln\left(\frac{x_i}{\beta_1}\right) - w\alpha_1 x_i^{\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}\ln(\beta_1)}{\beta_1^{\alpha_1}}\right)$$

$$\left(\frac{w\alpha_1^2 x_i^{\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}\left(\frac{x_i}{\beta_1}\right)^{\alpha_1} - w\alpha_1^2 x_i^{\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}}{\beta_1\beta_1^{\alpha_1}}\right)\Bigg)$$

8. Akan ditunjukkan turunan dari persamaan (2) pada halaman IV-4 terhadap β_1 sebagai berikut:

$$\frac{d2}{d\beta_1} = \sum_{i=1}^{n}\left(\frac{1}{\left(w\left(\frac{\alpha_1 x_i^{\alpha_1-1}e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right) + (1-w)\left(\frac{\alpha_2 (x_i-\gamma)^{\alpha_2-1}e^{-((x_i-\gamma)/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right)\right)}\right.$$

$$\left(\frac{2w\alpha_1^3 x_i^{\alpha_1-1} e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}} \left(\left(\frac{x_i}{\beta_1}\right)^{\alpha_1} \right)^2 - 3w\alpha_1^3 x_i^{\alpha_1-1} e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}} \left(\frac{x_i}{\beta_1}\right)^{\alpha_1} -}{\beta_1^2 \beta_1^{\alpha_1}} \right.$$

$$\left. \frac{w\alpha_1^2 x_i^{\alpha_1-1}.e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}} \left(\frac{x_i}{\beta_1}\right)^{\alpha_1} + w\alpha_1^3 x_i^{\alpha_1-1}.e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}} - w\alpha_1^2 x_i^{\alpha_1-1}.e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}}{\beta_1^2 \beta_1^{\alpha_1}} \right) \Bigg) -$$

$$\left(\frac{\left(\frac{w\alpha_1^2 x_i^{\alpha_1-1} e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}} \left(\frac{x_i}{\beta_1}\right)^{\alpha_1} - w\alpha_1^2 x_i^{\alpha_1-1} e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}}{\beta_1 \beta_1^{\alpha_1}} \right)^2}{\left(\frac{w\alpha_1 x_i^{\alpha_1-1} e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}}{\beta_1^{\alpha_1}} + \frac{(1-w)\alpha_2 (x_i-\gamma)^{\alpha_2-1} e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}}{\beta_2^{\alpha_2}} \right)^2} \right)$$

9. Akan ditunjukkan turunan dari persamaan (2) pada halaman IV-4 terhadap α_2 sebagai berikut:

$$\frac{d2}{d\alpha_2} = \sum_{i=1}^{n} \left(\frac{1}{\left(w\left(\frac{\alpha_1 x_i^{\alpha_1-1} e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}} \right) + (1-w)\left(\frac{\alpha_2 (x_i-\gamma)^{\alpha_2-1} e^{-(x_i-\gamma/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}} \right) \right)^2} \right.$$

$$\left(\frac{w\alpha_1^2 x_i^{\alpha_1-1} e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}} \left(\frac{x_i}{\beta_1}\right)^{\alpha_1} - w\alpha_1^2 x_i^{\alpha_1-1} e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}}{\beta_1 \beta_1^{\alpha_1}} \left(\frac{(1-w)(x_i-\gamma)^{\alpha_2-1}}{\beta_2^{\alpha_2}} \right. \right.$$

$$\frac{e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}} + (1-w)\alpha_2 (x_i-\gamma)^{\alpha_2-1}\ln(x_i-\gamma) e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}}{\beta_2^{\alpha_2}} -$$

$$\frac{(1-w)\alpha_2 (x_i-\gamma)^{\alpha_2-1} e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}} \left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2} \ln\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}{\beta_2^{\alpha_2}} -$$

$$\left.\left.\frac{(1-w)\alpha_2 (x_i-\gamma)^{\alpha_2-1} e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}} \ln(\beta_2)}{\beta_2^{\alpha_2}}\right)\right)$$

10. Akan ditunjukkan turunan dari persamaan (2) pada halaman IV-4 terhadap β_2 sebagai berikut:

$$\frac{d2}{d\beta_2} = \sum_{i=1}^{n} \left(\frac{1}{\left(w\left(\frac{\alpha_1 x_i^{\alpha_1-1} e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}} \right) + (1-w)\left(\frac{\alpha_2 (x_i-\gamma)^{\alpha_2-1} e^{-(x_i-\gamma/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}} \right) \right)^2} \right.$$

$$\left(\frac{w\alpha_1^2 x_i^{\alpha_1-1} e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}} \left(\frac{x_i}{\beta_1}\right)^{\alpha_1} - w\alpha_1^2 x_i^{\alpha_1-1} e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}}{\beta_1 \beta_1^{\alpha_1}} \right)$$

$$\left.\left(\frac{(1-w)\alpha_2^2 (x_i-\gamma)^{\alpha_2-1} e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}} \left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2} - (1-w)\alpha_2^2 (x_i-\gamma)^{\alpha_2-1} e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}}{\beta_2 \beta_2^{\alpha_2}} \right)\right)$$

11. Akan ditunjukkan turunan dari persamaan (2) pada halaman IV-4 terhadap γ sebagai berikut:

$$\frac{d2}{d\gamma}=\frac{1}{\left[w\left[\frac{\alpha_1 x_i^{\alpha_1-1}e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right]+(1-w)\left[\frac{\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-(x_i-\gamma/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right]\right]^2}$$

$$\left[\sum_{i=1}^{n}\left[\frac{w\alpha_1^2 x_i^{\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}\ln(e)-w\alpha_1^2 x_i^{\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}}{\beta_1\beta_1^{\alpha_1}}\right]\right.$$

$$\left[\frac{-(1-w)\alpha_2(x_i-\gamma)^{\alpha_2-1}(\alpha_2-1)e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}}{(x_i-\gamma)\beta_2^{\alpha_2}}\right.$$

$$\left.\left.\frac{+(1-w)\alpha_2^2(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}\ln(e)}{(x_i-\gamma)\beta_2^{\alpha_2}}\right]\right]$$

12. Akan ditunjukkan turunan dari persamaan (1) pada halaman IV-3 terhadap w sebagai berikut:

$$\frac{d2}{dw}=\sum_{i=1}^{n}\frac{\left[\frac{\alpha_1^2 x_i^{\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}\ln(e)-\alpha_1^2 x_i^{\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}}{\beta_1\beta_1^{\alpha_1}}\right]^2}{\left[\frac{w\alpha_1 x_i^{\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}}{\beta_1^{\alpha_1}}+\frac{(1-w)\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}}{\beta_2^{\alpha_2}}\right]^2}-$$

$$\frac{1}{\left[w\left[\frac{\alpha_1 x_i^{\,\alpha_1-1}e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right]+(1-w)\left[\frac{\alpha_2\,(x_i-\gamma)^{\alpha_2-1}\,e^{-(x_i-\gamma/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right]\right]^2}$$

$$\left[\sum_{i=1}^{n}\left[\frac{w\alpha_1^2 x_i^{\,\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}\ln(e)-w\alpha_1^2 x_i^{\,\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}}{\beta_1\beta_1^{\alpha_1}}\right]\right.$$

$$\left.\left[\frac{\alpha_1 x_i^{\,\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}}{\beta_1^{\alpha_1}}-\frac{\alpha_2\,(x_i-\gamma)^{\alpha_2-1}\,e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right]\right]$$

13. Akan ditunjukkan turunan dari persamaan (3) pada halaman IV-4 terhadap α_1 sebagai berikut:

$$\frac{d3}{d\alpha_1}=-\frac{1}{\left[w\left[\frac{\alpha_1 x_i^{\,\alpha_1-1}e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right]+(1-w)\left[\frac{\alpha_2\,(x_i-\gamma)^{\alpha_2-1}\,e^{-(x_i-\gamma/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right]\right]^2}$$

$$\left[\sum_{i=1}^{n}\left[\frac{wx_i^{\,\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}+w\alpha_1 x_i^{\,\alpha_1-1}\ln(x_i)e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}-w\alpha_1 x_i^{\,\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}\ln\beta_1-}{\beta_1^{\alpha_1}}\right.\right.$$

$$\left.\frac{w\alpha_1 x_i^{\,\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}\ln\left(\frac{x_i}{\beta_1}\right)\ln(e)}{\beta_1^{\alpha_1}}\right]\left[\frac{(1-w)(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha 2}}+}{\beta_2^{\alpha_2}}\right.$$

$$\frac{(1-w)\alpha_2\,(x_i-\gamma)^{\alpha_2-1}\ln(x_i-\gamma)e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha 2}}}{\beta_2^{\alpha_2}}$$

$$-\frac{(1-w)\alpha_2\,(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha 2}}\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}\ln\left(\frac{x_i-\gamma}{\beta_2}\right)\ln(e)}{\beta_2^{\alpha_2}}$$

$$-\frac{(1-w)\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\ln(\beta_2)}{\beta_2^{\alpha_2}}\Bigg]\Bigg]$$

14. Akan ditunjukkan turunan dari persamaan (3) pada halaman IV-4 terhadap β_1 sebagai berikut:

$$\frac{d3}{d\beta_1}=-\frac{1}{\left[w\left[\frac{\alpha_1 x_i^{\alpha_1-1}e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right]+(1-w)\left[\frac{\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-(x_i-\gamma/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right]\right]^2}$$

$$\left[\sum_{i=1}^{n}\left[\frac{w\alpha_1^2 x_i^{\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}\ln(e)-w\alpha_1^2 x_i^{\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}}{\beta_1\beta_1^{\alpha_1}}\right]-\right.$$

$$\left[\frac{(1-w)(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}+(1-w)\alpha_2(x_i-\gamma)^{\alpha_2-1}\ln(x_i-\gamma)e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}}{\beta_2^{\alpha_2}}\right.$$

$$-\frac{(1-w)\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}\ln\left(\frac{x_i-\gamma}{\beta_2}\right)\ln(e)}{\beta_2^{\alpha_2}}$$

$$-\frac{(1-w)\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\ln(\beta_2)}{\beta_2^{\alpha_2}}\Bigg]\Bigg]$$

15. Akan ditunjukkan turunan dari persamaan (3) pada halaman IV-4 terhadap α_2 sebagai berikut:

$$\frac{d3}{d\alpha_2}=\frac{1}{\left[w\left[\frac{\alpha_1 x_i^{\alpha_1-1}e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right]+(1-w)\left[\frac{\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-(x_i-\gamma/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right]\right]}$$

$$\left[\sum_{i=1}^{n}\left[\frac{2(1-w)(x_i-\gamma)^{\alpha_2-1}\ln(x_i-\gamma)e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}}{\beta_2^{\alpha_2}}\right.\right.$$

$$-\frac{2(1-w)(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}\ln\left(\frac{x_i-\gamma}{\beta_2}\right)\ln(e)-2(1-w)}{\beta_2^{\alpha_2}}$$

$$-\frac{2(1-w)\alpha_2(x_i-\gamma)^{\alpha_2-1}\ln(x_i-\gamma)e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}\ln\left(\frac{x_i-\gamma}{\beta_2}\right)\ln(e)}{\beta_2^{\alpha_2}}$$

$$-\frac{2(1-w)\alpha_2(x_i-\gamma)^{\alpha_2-1}\ln(x_i-\gamma)e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\ln(\beta_2)}{\beta_2^{\alpha_2}}$$

$$+\frac{(1-w)\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\left(\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}\right)^2\ln\left(\frac{x_i-\gamma}{\beta_2}\right)^2\ln(e)^2}{\beta_2^{\alpha_2}}$$

$$-\frac{(1-w)\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}\ln\left(\frac{x_i-\gamma}{\beta_2}\right)^2\ln(e)}{\beta_2^{\alpha_2}}$$

$$+\frac{2(1-w)\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}\ln\left(\frac{x_i-\gamma}{\beta_2}\right)\ln(e)\ln\beta_2}{\beta_2^{\alpha_2}}$$

$$\left.\left.+\frac{(1-w)\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\ln(\beta_2)^2}{\beta_2^{\alpha_2}}\right]\right]-$$

$$\frac{1}{\left[w\left[\frac{\alpha_1 x_i^{\alpha_1-1}e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right]+(1-w)\left[\frac{\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-(x_i-\gamma/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right]\right]^2}$$

$$\left[\sum_{i=1}^{n}\left[\frac{(1-w)(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}+(1-w)\alpha_2(x_i-\gamma)^{\alpha_2-1}\ln(x_i-\gamma)e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}}{\beta_2^{\alpha_2}}\right.\right.$$

$$-\frac{(1-w)\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}\ln\left(\frac{x_i-\gamma}{\beta_2}\right)\ln(e)}{\beta_2^{\alpha_2}}$$

$$\left.\left.-\frac{(1-w)\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\ln(\beta_2)}{\beta_2^{\alpha_2}}\right]^2\right]$$

16. Akan ditunjukkan turunan dari persamaan (3) pada halaman IV-4 terhadap β_2 sebagai berikut:

$$\frac{d3}{d\beta_2}=\frac{1}{\left[w\left[\frac{\alpha_1 x_i^{\alpha_1-1}e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right]+(1-w)\left[\frac{\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-(x_i-\gamma/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right]\right]}$$

$$\left[\sum_{i=1}^{n}\left[\frac{2(1-w)(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}\alpha_2\ln(e)}{\beta_2\beta_2^{\alpha_2}}\right.\right.$$

$$-\frac{2(1-w)(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\alpha_2}{\beta_2\beta_2^{\alpha_2}}$$

$$+\frac{(1-w)\alpha_2^2(x_i-\gamma)^{\alpha_2-1}\ln(x_i-\gamma)e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}\ln(e)}{\beta_2\beta_2^{\alpha_2}}$$

$$-\frac{(1-w)\alpha_2^2(x_i-\gamma)^{\alpha_2-1}\ln(x_i-\gamma)e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}}{\beta_2\beta_2^{\alpha_2}}$$

$$-\frac{(1-w)\alpha_2^2(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\left(\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}\right)^2\ln\left(\frac{x_i-\gamma}{\beta_2}\right)\ln(e)^2}{\beta_2\beta_2^{\alpha_2}}$$

$$+\frac{2(1-w)\alpha_2^2(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}\ln\left(\frac{x_i-\gamma}{\beta_2}\right)\ln(e)}{\beta_2\beta_2^{\alpha_2}}$$

$$-\frac{(1-w)\alpha_2^2(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}\ln(\beta_2)\ln(e)}{\beta_2\beta_2^{\alpha_2}}$$

$$\left.\left.+\frac{(1-w)\alpha_2^2(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}\ln(\beta_2)}{\beta_2\beta_2^{\alpha_2}}\right]\right]-$$

$$\frac{1}{\left[w\left[\frac{\alpha_1 x_i^{\alpha_1-1}e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right]+(1-w)\left[\frac{\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-(x_i-\gamma/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right]\right]^2}$$

$$\left[\sum_{i=1}^{n}\left[\frac{(1-w)(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}+(1-w)\alpha_2(x_i-\gamma)^{\alpha_2-1}\ln(x_i-\gamma)e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}}{\beta_2^{\alpha_2}}\right.\right.$$

$$-\frac{(1-w)\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}\ln\left(\frac{x_i-\gamma}{\beta_2}\right)\ln(e)}{\beta_2^{\alpha_2}}$$

$$\left.-\frac{(1-w)\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\ln(\beta_2)}{\beta_2^{\alpha_2}}\right]\left[\frac{(1-w)\alpha_2^2(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}\ln(e)}{\beta_2\beta_2^{\alpha_2}}\right.$$

$$\left.\left.+\frac{(1-w)\alpha_2^2(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}}{\beta_2\beta_2^{\alpha_2}}\right]\right]$$

17. Akan ditunjukkan turunan dari persamaan (3) pada halaman IV-4 terhadap γ sebagai berikut:

$$\frac{d3}{d\gamma}=\frac{1}{\left[w\left[\frac{\alpha_1 x_i^{\alpha_1-1}e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right]+(1-w)\left[\frac{\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-(x_i-\gamma/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right]\right]}$$

$$\left[\sum_{i=1}^{n}\left[-\frac{(1-w)(x_i-\gamma)^{\alpha_2-1}(\alpha_2-1)e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}}{(x_i-\gamma)\beta_2^{\alpha_2}}\right.\right.$$

$$+\frac{2(1-w)(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}\alpha_2\ln(e)}{(x_i-\gamma)\beta_2^{\alpha_2}}$$

$$+\frac{(1-w)\alpha_2(x_i-\gamma)^{\alpha_2-1}(\alpha_2-1)\ln(x_i-\gamma)e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}}{(x_i-\gamma)\beta_2^{\alpha_2}}$$

$$-\frac{(1-w)\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}}{(x_i-\gamma)\beta_2^{\alpha_2}}$$

$$+\frac{(1-w)\alpha_2^2(x_i-\gamma)^{\alpha_2-1}\ln(x_i-\gamma)e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}\ln(e)}{(x_i-\gamma)\beta_2^{\alpha_2}}$$

$$+\frac{(1-w)\alpha_2(x_i-\gamma)^{\alpha_2-1}(\alpha_2-1)e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}\ln\left(\frac{x_i-\gamma}{\beta_2}\right)\ln(e)}{(x_i-\gamma)\beta_2^{\alpha_2}}$$

$$+\frac{(1-w)\alpha_2^2(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\left(\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}\right)^2\ln\left(\frac{x_i-\gamma}{\beta_2}\right)\ln(e)^2}{(x_i-\gamma)\beta_2^{\alpha_2}}$$

$$+\frac{(1-w)\alpha_2^2(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}\ln\left(\frac{x_i-\gamma}{\beta_2}\right)\ln(e)}{(x_i-\gamma)\beta_2^{\alpha_2}}$$

$$+\frac{(1-w)\alpha_2(x_i-\gamma)^{\alpha_2-1}(\alpha_2-1)e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\ln(\beta_2)}{(x_i-\gamma)\beta_2^{\alpha_2}}$$

$$\left.\left.-\frac{(1-w)\alpha_2^2(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}\ln(\beta_2)\ln(e)}{(x_i-\gamma)\beta_2^{\alpha_2}}\right]\right]-$$

$$
\frac{1}{\left[w\left[\frac{\alpha_1 x_i^{\alpha_1-1} e^{-\left(x_i/\beta_1\right)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right]+(1-w)\left[\frac{\alpha_2 (x_i-\gamma)^{\alpha_2-1} e^{-\left(x_i-\gamma/\beta_2\right)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right]\right]^2}
$$

$$
\left[\sum_{i=1}^{n}\left[\frac{(1-w)(x_i-\gamma)^{\alpha_2-1} e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}+(1-w)\alpha_2 (x_i-\gamma)^{\alpha_2-1}\ln(x_i-\gamma)e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}}{\beta_2^{\alpha_2}}\right.\right.
$$

$$
-\frac{(1-w)\alpha_2 (x_i-\gamma)^{\alpha_2-1} e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}\ln\left(\frac{x_i-\gamma}{\beta_2}\right)\ln(e)}{\beta_2^{\alpha_2}}
$$

$$
\left.-\frac{(1-w)\alpha_2 (x_i-\gamma)^{\alpha_2-1} e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\ln(\beta_2)}{\beta_2^{\alpha_2}}\right]\left[-\frac{(1-w)\alpha_2 (x_i-\gamma)^{\alpha_2-1}(\alpha_2-1)e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}}{(x_i-\gamma)\beta_2^{\alpha_2}}\right.
$$

$$
\left.\left.+\frac{(1-w)\alpha_2^2 (x_i-\gamma)^{\alpha_2-1} e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}\ln(e)}{(x_i-\gamma)\beta_2^{\alpha_2}}\right]\right]
$$

18. Akan ditunjukkan turunan dari persamaan (3) pada halaman IV-4 terhadap w sebagai berikut:

$$
\frac{d3}{dw}=\frac{1}{\left[w\left[\frac{\alpha_1 x_i^{\alpha_1-1} e^{-\left(x_i/\beta_1\right)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right]+(1-w)\left[\frac{\alpha_2 (x_i-\gamma)^{\alpha_2-1} e^{-\left(x_i-\gamma/\beta_2\right)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right]\right]}
$$

$$
\left[\sum_{i=1}^{n}\left[-\frac{(x_i-\gamma)^{\alpha_2-1} e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}-\alpha_2 (x_i-\gamma)^{\alpha_2-1}\ln(x_i-\gamma)e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}}{\beta_2^{\alpha_2}}\right.\right.
$$

$$
+\frac{\alpha_2 (x_i-\gamma)^{\alpha_2-1} e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}\ln\left(\frac{x_i-\gamma}{\beta_2}\right)\ln(e)}{\beta_2^{\alpha_2}}+\frac{\alpha_2 (x_i-\gamma)^{\alpha_2-1} e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}}{\beta_2^{\alpha_2}}
$$

$$
\left.\left.\frac{\ln(\beta_2)}{\beta_2^{\alpha_2}}\right]\right] - \frac{1}{\left[w\left[\dfrac{\alpha_1 x_i^{\alpha_1-1} e^{-\left(x_i/\beta_1\right)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right] + (1-w)\left[\dfrac{\alpha_2 (x_i-\gamma)^{\alpha_2-1} e^{-\left(x_i-\gamma/\beta_2\right)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right]\right]^2}
$$

$$
\left[\sum_{i=1}^{n}\left[\frac{(1-w)(x_i-\gamma)^{\alpha_2-1} e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha 2}} + (1-w)\alpha_2 (x_i-\gamma)^{\alpha_2-1}\ln(x_i-\gamma) e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right.\right.
$$

$$
-\frac{(1-w)\alpha_2 (x_i-\gamma)^{\alpha_2-1} e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha 2}}\left(\dfrac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}\ln\left(\dfrac{x_i-\gamma}{\beta_2}\right)\ln(e)}{\beta_2^{\alpha_2}} - \frac{e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha 2}}\ln(\beta_2)}{\beta_2^{\alpha_2}}
$$

$$
\left.\left.\frac{e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha 2}}\ln(\beta_2)}{\beta_2^{\alpha_2}}\right]\right]\left[\left[\frac{\alpha_1 x_i^{\alpha_1-1} e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha 2}}}{\beta_1^{\alpha_1}} - \frac{\alpha_2 (x_i-\gamma)^{\alpha_2-1} e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right]\right]
$$

19. Akan ditunjukkan turunan dari persamaan (4) pada halaman IV-5 terhadap α_1 sebagai berikut:

$$
\frac{d4}{d\alpha_1} = -\frac{1}{\left[w\left[\dfrac{\alpha_1 x_i^{\alpha_1-1} e^{-\left(x_i/\beta_1\right)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right] + (1-w)\left[\dfrac{\alpha_2 (x_i-\gamma)^{\alpha_2-1} e^{-\left(x_i-\gamma/\beta_2\right)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right]\right]^2}
$$

$$
\left[\sum_{i=1}^{n}\left[\frac{w x_i^{\alpha_1-1} e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}} + w\alpha_1 x_i^{\alpha_1-1}\ln(x_i) e^{-\left(\frac{}{\beta_1}\right)^{\alpha_1}} - w\alpha_1 x_i^{\alpha_1-1} e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}\left(\dfrac{x_i}{\beta_1}\right)^{\alpha_1}\ln\left(\dfrac{x_i}{\beta_1}\right)\ln e}{\beta_1^{\alpha_1}}\right.\right.
$$

$$
\left.-\frac{w\alpha_1 x_i^{\alpha_1-1} e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}\ln(\beta_1)}{\beta_2\beta_2^{\alpha_2}}\right]\left[\frac{(1-w)\alpha_2^2 (x_i-\gamma)^{\alpha_2-1} e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha 2}}\left(\dfrac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}\ln(e)}{\beta_2\beta_2^{\alpha_2}}\right.
$$

$$
\left.\left.-\frac{(1-w)\alpha_2^2 (x_i-\gamma)^{\alpha_2-1} e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha 2}}}{\beta_2\beta_2^{\alpha_2}}\right]\right]
$$

20. Akan ditunjukkan turunan dari persamaan (4) pada halaman IV-5 terhadap β_1 sebagai berikut:

$$\frac{d4}{d\beta_1} = -\frac{1}{\left[w\left[\frac{\alpha_1 x_i^{\alpha_1 - 1} e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right] + (1-w)\left[\frac{\alpha_2 (x_i-\gamma)^{\alpha_2 - 1} e^{-(x_i-\gamma/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right]\right]^2}$$

$$\left[\sum_{i=1}^{n}\left[\frac{w\alpha_1^2 x_i^{\alpha_1-1} e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}} \left(\frac{x_i}{\beta_1}\right)^{\alpha_1} \ln(e) - w\alpha_1^2 x_i^{\alpha_1-1} e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}}{\beta_1 \beta_1^{\alpha_1}}\right]\right.$$

$$\left[\frac{(1-w)\alpha_2^2 (x_i-\gamma)^{\alpha_2-1} e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}} \left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2} \ln(e)}{\beta_2 \beta_2^{\alpha_2}}\right.$$

$$\left.\left.\frac{(1-w)\alpha_2^2 (x_i-\gamma)^{\alpha_2-1} e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}}{\beta_2 \beta_2^{\alpha_2}}\right]\right]$$

21. Akan ditunjukkan turunan dari persamaan (4) pada halaman IV-5 terhadap α_2 sebagai berikut:

$$\frac{d4}{d\alpha_2} = \frac{1}{\left[w\left[\frac{\alpha_1 x_i^{\alpha_1 - 1} e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right] + (1-w)\left[\frac{\alpha_2 (x_i-\gamma)^{\alpha_2 - 1} e^{-(x_i-\gamma/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right]\right]}$$

$$\left[\sum_{i=1}^{n}\left[\frac{2(1-w)(x_i-\gamma)^{\alpha_2-1} e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}} \left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2} \alpha_2 \ln(e)}{\beta_2 \beta_2^{\alpha_2}}\right]\right.$$

$$-\frac{2(1-w)(x_i-\gamma)^{\alpha_2-1} e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}} \alpha_2}{\beta_2 \beta_2^{\alpha_2}}$$

$$+\frac{(1-w)\alpha_2^2(x_i-\gamma)^{\alpha_2-1}\ln(x_i-\gamma)e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}\ln(e)}{\beta_2\beta_2^{\alpha_2}}$$

$$-\frac{(1-w)\alpha_2^2(x_i-\gamma)^{\alpha_2-1}\ln(x_i-\gamma)e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}}{\beta_2\beta_2^{\alpha_2}}$$

$$-\frac{(1-w)\alpha_2^2(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\left[\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}\right]^2\ln(e)^2\ln\left(\frac{x_i-\gamma}{\beta_2}\right)}{\beta_2\beta_2^{\alpha_2}}$$

$$+\frac{2(1-w)\alpha_2^2(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}\ln(e)^2\ln\left(\frac{x_i-\gamma}{\beta_2}\right)}{\beta_2\beta_2^{\alpha_2}}$$

$$-\frac{(1-w)\alpha_2^2(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}\ln(e)\ln(\beta_2)}{\beta_2\beta_2^{\alpha_2}}$$

$$\left.\left.+\frac{(1-w)\alpha_2^2(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\ln(\beta_2)}{\beta_2\beta_2^{\alpha_2}}\right]\right]-$$

$$\frac{1}{\left[w\left[\frac{\alpha_1 x_i^{\alpha_1-1}e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right]+(1-w)\left[\frac{\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-(x_i-\gamma/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right]\right]^2}$$

$$\left[\sum_{i=1}^{n}\left[\frac{(1-w)(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}+(1-w)\alpha_2(x_i-\gamma)^{\alpha_2-1}\ln(x_i-\gamma)e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}}{\beta_2^{\alpha_2}}\right.\right.$$

$$-\frac{(1-w)\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}\ln\left(\frac{x_i-\gamma}{\beta_2}\right)\ln(e)}{\beta_2^{\alpha_2}}$$

$$-\frac{(1-w)\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\ln(\beta_2)}{\beta_2^{\alpha_2}}\left]\left[\frac{(1-w)\alpha_2^2(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}\ln(\beta_2)}{\beta_2\beta_2^{\alpha_2}}\right.\right.$$

$$-\frac{(1-w)\alpha_2^2(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}}{\beta_2\beta_2^{\alpha_2}}$$

22. Akan ditunjukkan turunan dari persamaan (4) pada halaman IV-5 terhadap β_2 sebagai berikut:

$$\frac{d4}{d\beta_2}=\frac{1}{\left[w\left[\frac{\alpha_1 x_i^{\alpha_1-1}e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right]+(1-w)\left[\frac{\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-(x_i-\gamma/\beta_2)^{\alpha_2}}}{\beta_2^{\alpha_2}}\right]\right]}$$

$$\left[\sum_{i=1}^{n}\left[\frac{(1-w)\alpha_2^3(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\left(\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}\right)^2\ln(e)^2}{\beta_2^2\beta_2^{\alpha_2}}\right.\right.$$

$$-\frac{3(1-w)\alpha_2^3(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}\ln(e)}{\beta_2^2\beta_2^{\alpha_2}}$$

$$-\frac{(1-w)\alpha_2^2(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}\ln(e)}{\beta_2^2\beta_2^{\alpha_2}}$$

$$\left.\left.+\frac{(1-w)\alpha_2^3(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}+(1-w)\alpha_2^2(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}}{\beta_2^2\beta_2^{\alpha_2}}\right]\right]-$$

$$\frac{1}{\left[w\left[\frac{\alpha_1 x_i^{\alpha_1-1} e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right]+(1-w)\left[\frac{\alpha_2 (x_i-\gamma)^{\alpha_2-1} e^{-(x_i-\gamma/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right]\right]^2}$$

$$\left[\sum_{i=1}^{n}\left[\frac{(1-w)\alpha_2^2 (x_i-\gamma)^{\alpha_2-1} e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}\ln(e)}{\beta_2\beta_2^{\alpha_2}}\right.\right.$$

$$\left.\left.-\frac{(1-w)\alpha_2^2 (x_i-\gamma)^{\alpha_2-1} e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}}{\beta_2\beta_2^{\alpha_2}}\right]^2\right]$$

23. Akan ditunjukkan turunan dari persamaan (4) pada halaman IV-5 terhadap γ sebagai berikut:

$$\frac{d4}{d\gamma}=\frac{1}{\left[w\left[\frac{\alpha_1 x_i^{\alpha_1-1} e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right]+(1-w)\left[\frac{\alpha_2 (x_i-\gamma)^{\alpha_2-1} e^{-(x_i-\gamma/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right]\right]}$$

$$\left[\sum_{i=1}^{n}\left[-\frac{(1-w)\alpha_2^2 (x_i-\gamma)^{\alpha_2-1}(\alpha_2-1) e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}\ln(e)}{(x_i-\gamma)\beta_2\beta_2^{\alpha_2}}\right.\right.$$

$$+\frac{(1-w)\alpha_2^3 (x_i-\gamma)^{\alpha_2-1} e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\left(\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}\right)^2\ln(e)^2}{(x_i-\gamma)\beta_2\beta_2^{\alpha_2}}$$

$$-\frac{2(1-w)\alpha_2^3 (x_i-\gamma)^{\alpha_2-1} e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}\ln(e)}{(x_i-\gamma)\beta_2\beta_2^{\alpha_2}}$$

$$\left.\left.+\frac{(1-w)\alpha_2^2 (x_i-\gamma)^{\alpha_2-1}(\alpha_2-1) e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}}{(x_i-\gamma)\beta_2\beta_2^{\alpha_2}}\right]\right]-$$

$$\frac{1}{\left[w\left[\frac{\alpha_1 x_i^{\alpha_1-1} e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right]+(1-w)\left[\frac{\alpha_2 (x_i-\gamma)^{\alpha_2-1} e^{-(x_i-\gamma/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right]\right]^2}$$

$$\left[\sum_{i=1}^{n}\left[\frac{(1-w)\alpha_2^2 (x_i-\gamma)^{\alpha_2-1} e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha 2}} \left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2} \ln(e)}{\beta_2 \beta_2^{\alpha_2}}\right.\right.$$

$$\left.\left. -\frac{(1-w)\alpha_2^2 (x_i-\gamma)^{\alpha_2-1} e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha 2}}}{\beta_2 \beta_2^{\alpha_2}}\right]\right]\left[-\frac{(1-w)\alpha_2 (x_i-\gamma)^{\alpha_2-1}(\alpha_2-1) e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha 2}}}{(x_i-\gamma)\beta_2 \beta_2^{\alpha_2}}\right.$$

$$\left.\left. +\frac{(1-w)\alpha_2^2 (x_i-\gamma)^{\alpha_2-1} e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha 2}} \left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2} \ln(e)}{(x_i-\gamma)\beta_2^{\alpha_2}}\right]\right]$$

24. Akan ditunjukkan turunan dari persamaan (4) pada halaman IV-5 terhadap w sebagai berikut:

$$\frac{d4}{dw}=\sum_{i=1}^{n}\frac{\left[-\frac{\alpha_2^2 (x_i-\gamma)^{\alpha_2-1} e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha 2}} \left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2} \ln(e)+\alpha_2^2 (x_i-\gamma)^{\alpha_2-1} e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha 2}}}{\beta_2 \beta_2^{\alpha_2}}\right]}{\left[\frac{w\alpha_1 x_i^{\alpha_1-1} e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha 1}}}{\beta_1^{\alpha_1}}+\frac{(1-w)\alpha_2 (x_i-\gamma)^{\alpha_2-1} e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right]^2}-$$

$$\frac{1}{\left[w\left[\frac{\alpha_1 x_i^{\alpha_1-1} e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right]+(1-w)\left[\frac{\alpha_2 (x_i-\gamma)^{\alpha_2-1} e^{-(x_i-\gamma/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right]\right]^2}$$

$$\left[\sum_{i=1}^{n}\left[\frac{(1-w)\alpha_2^2(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}\ln(e)-(1-w)\alpha_2^2}{\beta_2\beta_2^{\alpha_2}}\right.\right.$$

$$\left.\left.\frac{(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}}{\beta_2\beta_2^{\alpha_2}}\right]\left[-\frac{\alpha_1 x_i^{\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}}{\beta_1^{\alpha_1}}+\frac{\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}}{\beta_2^{\alpha_2}}\right]\right]$$

25. Akan ditunjukkan turunan dari persamaan (5) pada halaman IV-5 terhadap α_1 sebagai berikut:

$$\frac{d5}{d\alpha_1}=-\frac{1}{\left[w\left[\frac{\alpha_1 x_i^{\alpha_1-1}e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right]+(1-w)\left[\frac{\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-(x_i-\gamma/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right]\right]^2}$$

$$\left[\sum_{i=1}^{n}\left[\frac{wx_i^{\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}+w\alpha_1 x_i^{\alpha_1-1}\ln(x_i)e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right.\right.$$

$$\left.\frac{-w\alpha_1 x_i^{\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}\ln\left(\frac{x_i}{\beta_1}\right)\ln(e)-w\alpha_1 x_i^{\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}\ln\beta_1}{\beta_1^{\alpha_1}}\right]$$

$$\left[-\frac{(1-w)\alpha_2(x_i-\gamma)^{\alpha_2-1}(\alpha_2-1)e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}}{(x_i-\gamma)\beta_2\beta_2^{\alpha_2}}\right.$$

$$\left.\left.+\frac{(1-w)\alpha_2^2(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}\ln(e)}{(x_i-\gamma)\beta_2^{\alpha_2}}\right]\right]$$

26. Akan ditunjukkan turunan dari persamaan (5) pada halaman IV-5 terhadap β_1 sebagai berikut:

$$\frac{d5}{d\beta_1} = -\frac{1}{\left[w\left[\frac{\alpha_1 x_i^{\alpha_1-1} e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right] + (1-w)\left[\frac{\alpha_2 (x_i-\gamma)^{\alpha_2-1} e^{-(x_i-\gamma/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right]\right]^2}$$

$$\left[\sum_{i=1}^{n}\left[\frac{w\alpha_1^2 x_i^{\alpha_1-1} e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}\ln(e) - w\alpha_1^2 x_i^{\alpha_1-1} e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}}{\beta_1\beta_1^{\alpha_1}}\right]\right.$$

$$\left[-\frac{(1-w)\alpha_2 (x_i-\gamma)^{\alpha_2-1}(\alpha_2-1)e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha 2}}}{(x_i-\gamma)\beta_2^{\alpha_2}}\right.$$

$$\left.\left.+\frac{(1-w)\alpha_2^2 (x_i-\gamma)^{\alpha_2-1} e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha 2}}\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}\ln(e)}{(x_i-\gamma)\beta_2^{\alpha_2}}\right]\right]$$

27. Akan ditunjukkan turunan dari persamaan (5) pada halaman IV-5 terhadap α_2 sebagai berikut:

$$\frac{d5}{d\alpha_2} = \frac{1}{\left[w\left[\frac{\alpha_1 x_i^{\alpha_1-1} e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right] + (1-w)\left[\frac{\alpha_2 (x_i-\gamma)^{\alpha_2-1} e^{-(x_i-\gamma/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right]\right]}$$

$$\left[\sum_{i=1}^{n}\left[-\frac{(1-w)(x_i-\gamma)^{\alpha_2-1}(\alpha_2-1)e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha 2}}}{(x_i-\gamma)\beta_2^{\alpha_2}}\right.\right.$$

$$+\frac{2(1-w)(x_i-\gamma)^{\alpha_2-1} e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha 2}}\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}\alpha_2\ln(e) - (1-w)\alpha_2(x_i-\gamma)^{\alpha_2-1}}{(x_i-\gamma)\beta_2^{\alpha_2}}$$

$$\frac{(\alpha_2-1)\ln(x_i-\gamma)e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}-(1-w)\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}}{(x_i-\gamma)\beta_2^{\alpha_2}}$$

$$+\frac{(1-w)\alpha_2^2(x_i-\gamma)^{\alpha_2-1}\ln(x_i-\gamma)e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}\ln(e)}{(x_i-\gamma)\beta_2^{\alpha_2}}$$

$$+\frac{(1-w)\alpha_2(x_i-\gamma)^{\alpha_2-1}(\alpha_2-1)e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}\ln\left(\frac{x_i-\gamma}{\beta_2}\right)\ln(e)}{(x_i-\gamma)\beta_2^{\alpha_2}}$$

$$+\frac{(1-w)\alpha_2^2(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\left(\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}\right)^2\ln\left(\frac{x_i-\gamma}{\beta_2}\right)\ln(e)^2}{(x_i-\gamma)\beta_2^{\alpha_2}}$$

$$+\frac{(1-w)\alpha_2(x_i-\gamma)^{\alpha_2-1}(\alpha_2-1)e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\ln(\beta_2)}{(x_i-\gamma)\beta_2^{\alpha_2}}$$

$$\left.\left.-\frac{(1-w)\alpha_2^2(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}\ln(e)\ln(\beta_2)}{(x_i-\gamma)\beta_2^{\alpha_2}}\right]\right]-$$

$$\frac{1}{\left[w\left[\dfrac{\alpha_1 x_i^{\alpha_1-1}e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right]+(1-w)\left[\dfrac{\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-(x_i-\gamma/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right]\right]^2}$$

$$\left[\sum_{i=1}^{n}\left[\frac{(1-w)(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}+(1-w)\alpha_2(x_i-\gamma)^{\alpha_2-1}\ln(x_i-\gamma)e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}}{\beta_2^{\alpha_2}}\right.\right.$$

$$-\frac{(1-w)\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}\ln\left(\frac{x_i-\gamma}{\beta_2}\right)\ln(e)}{\beta_2^{\alpha_2}}$$

$$\left.-\frac{(1-w)\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\ln(\beta_2)}{\beta_2^{\alpha_2}}\right]$$

$$\left[-\frac{(1-w)\alpha_2(x_i-\gamma)^{\alpha_2-1}(\alpha_2-1)e^{-\left(\frac{x-\gamma}{\beta_2}\right)^{\alpha_2}}}{(x-\gamma)\beta_2^{\alpha_2}}+\right.$$

$$\left.+\frac{(1-w)\alpha_2^2(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}\ln(e)}{(x_i-\gamma)\beta_2^{\alpha_2}}\right]\Bigg]$$

28. Akan ditunjukkan turunan dari persamaan (5) pada halaman IV-5 terhadap β_2 sebagai berikut:

$$\frac{d5}{d\beta_2}=\frac{1}{\left[w\left[\frac{\alpha_1 x_i^{\alpha_1-1}e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right]+(1-w)\left[\frac{\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-(x_i-\gamma/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right]\right]}$$

$$\left[\sum_{i=1}^{n}\left[-\frac{(1-w)\alpha_2^2(x_i-\gamma)^{\alpha_2-1}(\alpha_2-1)e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}\ln(e)}{(x_i-\gamma)\beta_2\beta_2^{\alpha_2}}\right.\right.$$

$$+\frac{(1-w)\alpha_2^3(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\left(\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}\right)^2\ln(e)^2}{(x_i-\gamma)\beta_2\beta_2^{\alpha_2}}$$

$$-\frac{2(1-w)\alpha_2^3(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}\ln(e)}{(x_i-\gamma)\beta_2\beta_2^{\alpha_2}}$$

$$\left.\left.+\frac{(1-w)\alpha_2^2(x_i-\gamma)^{\alpha_2-1}(\alpha_2-1)e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}}{(x_i-\gamma)\beta_2\beta_2^{\alpha_2}}\right]\right]-$$

$$\frac{1}{\left[w\left[\frac{\alpha_1 x_i^{\alpha_1-1}e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right]+(1-w)\left[\frac{\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-(x_i-\gamma/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right]\right]^2}$$

$$\left[\sum_{i=1}^{n}\left[\frac{(1-w)\alpha_2^2(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}\ln(e)}{(x_i-\gamma)\beta_2\beta_2^{\alpha_2}}\right.\right.$$

$$\left.\left.-\frac{(1-w)\alpha_2^2(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}}{\beta_2\beta_2^{\alpha_2}}\right]\right]\left[-\frac{(1-w)\alpha_2(x_i-\gamma)^{\alpha_2-1}(\alpha_2-1)e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}}{(x_i-\gamma)\beta_2^{\alpha_2}}\right.$$

$$\left.\left.+\frac{(1-w)\alpha_2^2(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}\ln(e)}{(x_i-\gamma)\beta_2^{\alpha_2}}\right]\right]$$

29. Akan ditunjukkan turunan dari persamaan (5) pada halaman IV-5 terhadap γ sebagai berikut:

$$\frac{d5}{d\gamma}=\frac{1}{\left[w\left[\frac{\alpha_1 x_i^{\alpha_1-1}e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right]+(1-w)\left[\frac{\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-(x_i-\gamma/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right]\right]}$$

$$\left[\sum_{i=1}^{n}\left[\frac{(1-w)\alpha_2(x_i-\gamma)^{\alpha_2-1}(\alpha_2-1)^2 e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}}{(x_i-\gamma)^2\beta_2^{\alpha_2}}\right.\right.$$

$$\frac{(1-w)\alpha_2(x_i-\gamma)^{\alpha_2-1}(\alpha_2-1)e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}}{(x_i-\gamma)^2\beta_2^{\alpha_2}}$$

$$-\frac{2(1-w)\alpha_2^2(x_i-\gamma)^{\alpha_2-1}(\alpha_2-1)e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}\ln(e)}{(x_i-\gamma)^2\beta_2^{\alpha_2}}$$

$$+\frac{(1-w)\alpha_2^3(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\left(\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}\right)^2\ln(e)^2}{(x_i-\gamma)^2\beta_2^{\alpha_2}}$$

$$
-\frac{(1-w)\alpha_2^3 (x_i-\gamma)^{\alpha_2-1} e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}} \left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2} \ln(e)}{(x_i-\gamma)^2 \beta_2^{\alpha_2}}
$$

$$
\left.\left. + \frac{(1-w)\alpha_2^2 (x_i-\gamma)^{\alpha_2-1} e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}} \left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2} \ln(e)}{(x_i-\gamma)^2 \beta_2^{\alpha_2}} \right]\right] -
$$

$$
\frac{1}{\left[w\left[\frac{\alpha_1 x_i^{\alpha_1-1} e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right] + (1-w)\left[\frac{\alpha_2 (x_i-\gamma)^{\alpha_2-1} e^{-(x_i-\gamma/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right]\right]^2}
$$

$$
\left[\sum_{i=1}^{n}\left[-\frac{(1-w)\alpha_2 (x_i-\gamma)^{\alpha_2-1}(\alpha_2-1) e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}}{(x_i-\gamma)\beta_2^{\alpha_2}} \right.\right.
$$

$$
\left.\left. \frac{(1-w)\alpha_2^2 (x_i-\gamma)^{\alpha_2-1} e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}} \left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2} \ln(e)}{(x_i-\gamma)\beta_2^{\alpha_2}} \right]^2 \right]
$$

30. Akan ditunjukkan turunan dari persamaan (5) pada halaman IV-5 terhadap w sebagai berikut:

$$
\frac{d5}{dw} = \frac{1}{\left[w\left[\frac{\alpha_1 x_i^{\alpha_1-1} e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right] + (1-w)\left[\frac{\alpha_2 (x_i-\gamma)^{\alpha_2-1} e^{-(x_i-\gamma/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right]\right]}
$$

$$
\left[\sum_{i=1}^{n}\left[\frac{\alpha_2 (x_i-\gamma)^{\alpha_2-1}(\alpha_2-1) e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}} - \alpha_2^2 (x_i-\gamma)^{\alpha_2-1} e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}} \left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2} \ln(e)}{(x_i-\gamma)\beta_2^{\alpha_2}}\right]\right]
$$

$$\frac{1}{\left[w\left[\frac{\alpha_1 x_i^{\alpha_1-1}e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right]+(1-w)\left[\frac{\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-(x_i-\gamma/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right]\right]^2}$$

$$\left[\sum_{i=1}^{n}\left[-\frac{(1-w)\alpha_2(x_i-\gamma)^{\alpha_2-1}(\alpha_2-1)e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}}{(x_i-\gamma)\beta_2^{\alpha_2}}\right.\right.$$

$$\left.\frac{(1-w)\alpha_2^2(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}\ln(e)}{(x_i-\gamma)\beta_2^{\alpha_2}}\right]$$

$$\left.\left[\frac{\alpha_1 x_i^{\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}}{\beta_1^{\alpha_1}}-\frac{\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}}{\beta_2^{\alpha_2}}\right]\right]$$

31. Akan ditunjukkan turunan dari persamaan (6) pada halaman IV-6 terhadap α_1 sebagai berikut:

$$\frac{d6}{d\alpha_1}=\frac{1}{\left[w\left[\frac{\alpha_1 x_i^{\alpha_1-1}e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right]+(1-w)\left[\frac{\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-(x_i-\gamma/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right]\right]}$$

$$\left[\sum_{i=1}^{n}\left[\frac{x_i^{\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}+\alpha_1 x^{\alpha_1-1}\ln(x_i)e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right.\right.$$

$$\left.\left.\frac{-\alpha_1 x_i^{\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}\ln\left(\frac{x_i}{\beta_1}\right)\ln(e)-\alpha_1 x_i^{\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}\ln\beta_1}{\beta_1^{\alpha_1}}\right]\right]-$$

$$\frac{1}{\left[w\left[\frac{\alpha_1 x_i^{\alpha_1-1} e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right]+(1-w)\left[\frac{\alpha_2 (x_i-\gamma)^{\alpha_2-1} e^{-((x_i-\gamma)/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right]\right]^2}$$

$$\left[\sum_{i=1}^{n}\left[\frac{w x_i^{\alpha_1-1} e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}} + w\alpha_1 x_i^{\alpha_1-1}\ln(x_i) e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right.\right.$$

$$\left.\frac{-w\alpha_1 x_i^{\alpha_1-1} e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}\ln\left(\frac{x_i}{\beta_1}\right)\ln(e) - w\alpha_1 x_i^{\alpha_1-1} e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}\ln\beta_1}{\beta_1^{\alpha_1}}\right]$$

$$\left.\left[\frac{\alpha_1 x_i^{\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}}{\beta_1^{\alpha_1}} - \frac{\alpha_2 (x_i-\gamma)^{\alpha_2-1} e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right]\right]$$

32. Akan ditunjukkan turunan dari persamaan (6) pada halaman IV-6 terhadap β_1 sebagai berikut:

$$\frac{d6}{d\beta_1}=\sum_{i=1}^{n}\frac{\left[\frac{\alpha_1^2 x_i^{\alpha_1-1} e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}\ln(e)-\alpha_1^2 x_i^{\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}}{\beta_1\beta_1^{\alpha_1}}\right]}{\left[\frac{w\alpha_1 x_i^{\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}}{\beta_1^{\alpha_1}}+\frac{(1-w)\alpha_2 (x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right]^2}-$$

$$\frac{1}{\left[w\left[\frac{\alpha_1 x_i^{\alpha_1-1} e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right]+(1-w)\left[\frac{\alpha_2 (x_i-\gamma)^{\alpha_2-1} e^{-((x_i-\gamma)/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}}\right]\right]^2}$$

$$\left[\sum_{i=1}^{n}\left[\frac{w\alpha_1^2 x_i^{\alpha_1-1} e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}\ln(e)-w\alpha_1^2 x_i^{\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}}{\beta_1\beta_1^{\alpha_1}}\right]\right.$$

$$\left[\frac{\alpha_1 x_i^{\alpha_1 - 1} e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}}{\beta_1^{\alpha_1}} - \frac{\alpha_2 (x_i - \gamma)^{\alpha_2 - 1} e^{-\left(\frac{x_i - \gamma}{\beta_2}\right)^{\alpha_2}}}{\beta_2^{\alpha_2}}\right]\Bigg]$$

33. Akan ditunjukkan turunan dari persamaan (6) pada halaman IV-6 terhadap α_2 sebagai berikut:

$$\frac{d6}{d\alpha_2} = \frac{1}{\left[w\left[\frac{\alpha_1 x_i^{\alpha_1 - 1} e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right] + (1-w)\left[\frac{\alpha_2 (x_i - \gamma)^{\alpha_2 - 1} e^{-(x_i - \gamma/\beta_2)^{\alpha_2}}}{\beta_2^{\alpha_2}}\right]\right]}$$

$$\left[\sum_{i=1}^{n}\left[-\frac{(x_i - \gamma)^{\alpha_2 - 1} e^{-\left(\frac{x_i - \gamma}{\beta_2}\right)^{\alpha_2}} - \alpha_2 (x_i - \gamma)^{\alpha_2 - 1} \ln(x_i - \gamma) e^{-\left(\frac{x_i - \gamma}{\beta_2}\right)^{\alpha_2}}}{\beta_2^{\alpha_2}}\right.\right.$$

$$+ \frac{\alpha_2 (x_i - \gamma)^{\alpha_2 - 1} e^{-\left(\frac{x_i - \gamma}{\beta_2}\right)^{\alpha_2}} \left(\frac{x_i - \gamma}{\beta_2}\right)^{\alpha_2} \ln\left(\frac{x_i - \gamma}{\beta_2}\right) \ln(e)}{\beta_2^{\alpha_2}}$$

$$\left.\left. + \frac{\alpha_2 (x_i - \gamma)^{\alpha_2 - 1} e^{-\left(\frac{x_i - \gamma}{\beta_2}\right)^{\alpha_2}} \ln(\beta_2)}{\beta_2^{\alpha_2}}\right]\right] -$$

$$\frac{1}{\left[w\left[\frac{\alpha_1 x_i^{\alpha_1 - 1} e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right] + (1-w)\left[\frac{\alpha_2 (x_i - \gamma)^{\alpha_2 - 1} e^{-(x_i - \gamma/\beta_2)^{\alpha_2}}}{\beta_2^{\alpha_2}}\right]\right]^2}$$

$$\left[\sum_{i=1}^{n}\left[\frac{(1-w)(x_i - \gamma)^{\alpha_2 - 1} e^{-\left(\frac{x_i - \gamma}{\beta_2}\right)^{\alpha_2}} + (1-w)\alpha_2 (x_i - \gamma)^{\alpha_2 - 1} \ln(x_i - \gamma) e^{-\left(\frac{x_i - \gamma}{\beta_2}\right)^{\alpha_2}}}{\beta_2^{\alpha_2}}\right.\right.$$

$$- \frac{(1-w)\alpha_2 (x_i - \gamma)^{\alpha_2 - 1} e^{-\left(\frac{x_i - \gamma}{\beta_2}\right)^{\alpha_2}} \left(\frac{x_i - \gamma}{\beta_2}\right)^{\alpha_2} \ln\left(\frac{x_i - \gamma}{\beta_2}\right) \ln(e)}{\beta_2^{\alpha_2}}$$

$$-\frac{(1-w)\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\ln(\beta_2)}{\beta_2^{\alpha_2}}\left]\left[\frac{\alpha_1 x_i^{\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}}{\beta_1^{\alpha_1}}-\frac{\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}}{\beta_2^{\alpha_2}}\right]\right]$$

34. Akan ditunjukkan turunan dari persamaan (6) pada halaman IV-6 terhadap β_2 sebagai berikut:

$$\frac{d6}{d\beta_2}=\sum_{i=1}^{n}\left(\frac{\left(-\frac{\alpha_2^2(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}+\alpha_2^2(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}}{\beta_2\beta_2^{\alpha_2}}\right)}{\left(\frac{w\alpha_1 x_i^{\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}}{\beta_1^{\alpha_1}}+\frac{(1-w)\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}}{\beta_2^{\alpha_2}}\right)^2}-\right.$$

$$\frac{1}{\left(w\left(\frac{\alpha_1 x_i^{\alpha_1-1}e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}}\right)+(1-w)\left(\frac{\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-(x_i-\gamma/\beta_2)^{\alpha_2}}}{\beta_2^{\alpha_2}}\right)\right)^2}$$

$$\left(\frac{(1-w)\alpha_2^2(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}{\beta_2\beta_2^{\alpha_2}}-\frac{(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\ln(\beta_2)}{\beta_2^{\alpha_2}}\right.$$

$$\left.\left.\frac{(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}\ln(\beta_2)}{\beta_2^{\alpha_2}}\right)\left(\frac{\alpha_1 x_i^{\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}}{\beta_1^{\alpha_1}}-\frac{\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}}{\beta_2^{\alpha_2}}\right)\right)$$

35. Akan ditunjukkan turunan dari persamaan (6) pada halaman IV-6 terhadap γ sebagai berikut:

$$\frac{d6}{d\gamma} = \sum_{i=1}^{n} \left(\frac{1}{\left(w\left(\frac{\alpha_1 x_i^{\alpha_1 - 1} e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}} \right) + (1-w)\left(\frac{\alpha_2 (x_i - \gamma)^{\alpha_2 - 1} e^{-(x_i-\gamma/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}} \right) \right)} \right.$$

$$\left(\frac{\alpha_2 (x_i - \gamma)^{\alpha_2 - 1} (\alpha_2 - 1) e^{-\left(\frac{x_i - \gamma}{\beta_2}\right)^{\alpha 2}} - \alpha_2^2 (x_i - \gamma)^{\alpha_2 - 1} e^{-\left(\frac{x_i - \gamma}{\beta_2}\right)^{\alpha 2}} \left(\frac{x_i - \gamma}{\beta_2} \right)^{\alpha_2}}{(x_i - \gamma)\beta_2^{\alpha_2}} \right)$$

$$\frac{1}{\left(w\left(\frac{\alpha_1 x_i^{\alpha_1 - 1} e^{-(x_i/\beta_1)^{\alpha_1}}}{\beta_1^{\alpha_1}} \right) + (1-w)\left(\frac{\alpha_2 (x_i - \gamma)^{\alpha_2 - 1} e^{-(x_i-\gamma/\beta_2)^{\alpha 2}}}{\beta_2^{\alpha_2}} \right) \right)^2}$$

$$\left(\left(-\frac{(1-w)\alpha_2 (x-\gamma)^{\alpha_2 - 1} (\alpha_2 - 1) e^{-\left(\frac{x-\gamma}{\beta_2}\right)^{\alpha 2}}}{(x-\gamma)\beta_2^{\alpha_2}} \right) \right.$$

$$\left. \frac{(1-w)\alpha_2^2 (x_i - \gamma)^{\alpha_2 - 1} e^{-\left(\frac{x_i - \gamma}{\beta_2}\right)^{\alpha 2}} \left(\frac{x_i - \gamma}{\beta_2} \right)^{\alpha_2}}{(x_i - \gamma)\beta_2^{\alpha_2}} \right)$$

$$\left. \left(\frac{\alpha_1 x_i^{\alpha_1 - 1} e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}}{\beta_1^{\alpha_1}} - \frac{\alpha_2 (x_i - \gamma)^{\alpha_2 - 1} e^{-\left(\frac{x_i - \gamma}{\beta_2}\right)^{\alpha 2}}}{\beta_2^{\alpha_2}} \right) \right)$$

36. Akan ditunjukkan turunan dari persamaan (6) pada halaman IV-6 terhadap w sebagai berikut:

$$\frac{d4}{dw}=\sum_{i=1}^{n}\left(-\frac{\left(\frac{\alpha_1 x_i^{\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}}{\beta_1^{\alpha_1}}-\frac{\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}}{\beta_2^{\alpha_2}}\right)^2}{\left(\frac{w\alpha_1 x_i^{\alpha_1-1}e^{-\left(\frac{x_i}{\beta_1}\right)^{\alpha_1}}}{\beta_1^{\alpha_1}}+\frac{(1-w)\alpha_2(x_i-\gamma)^{\alpha_2-1}e^{-\left(\frac{x_i-\gamma}{\beta_2}\right)^{\alpha_2}}}{\beta_2^{\alpha_2}}\right)^2}\right)$$

4.3 Aplikasi

Selanjutnya akan ditentukan nilai awal dari masing-masing parameter yaitu $\alpha_1,\beta_1,\alpha_2,\beta_2,\gamma,w$ dengan menggunakan data elektronik sebagai berikut:

Table 4.1 Data 20 *lifetimes of Elektronik*

i	x
1	0.03
2	0.12
3	0.22
4	0.35
5	0.73
6	0.79
7	1.25
8	1.41
9	1,52
10	1,79
11	1,80
12	1,94
13	2,38
14	2,40
15	2,87
16	2,99
17	3,14
18	3,17
19	4,72
20	5,09

data yang digunakan untuk penentuan nilai awal tersebut dibagi kedalam dua kelompok, berdasarkan nilai $w=0{,}4$ dan $\gamma=1$, yaitu:

Table 4.2 Data *lifetimes of 8 Elektronik*

i	x
1	0.03
2	0.12
3	0.22
4	0.35
5	0.73
6	0.79
7	1.25
8	1.41

Tabel 4.3 Data *lifetimes of 12 Elektronik*

i	x
1	1,52
2	1,79
3	1,80
4	1,94
5	2,38
6	2,40
7	2,87
8	2,99
9	3,14
10	3,17
11	4,72
12	5,09

Kelompok data pertama digunakan untuk distribusi Weibull berparameter dua, yaitu α_1 dan β_1. Selanjutnya data tersebut digunakan untuk menentukan nilai awal yang dapat ditunjukkan sebagai berikut:

Berdasarkan persamaan (2.2.12) maka:

$$F(x) = 1 - e^{-\left(x/\beta\right)^{\alpha}}$$

$$e^{-\left(x/\beta_1\right)^{\alpha_1}} = 1 - F(x)$$

$$\log\left(e^{-\left(x/\beta_1\right)^{\alpha_1}}\right) = \log(1 - F(x))$$

$$\log\left(e^{-\left(x/\beta_1\right)^{\alpha_1}}\right) = \log(\log(1 - F(x)))$$

$$-\alpha_1 \log\left(x/\beta_1\right) = \log(\log(1 - F(x)))$$

$$-\alpha_1(\log x - \log \beta_1) = \log(\log(1 - F(x)))$$

$$-(\log x - \log \beta_1) = \frac{1}{\alpha_1}\log(\log(1-F(x)))$$

$$\log x = \log \beta_1 + \frac{1}{\alpha_1}\log\left(\log\left(\frac{1}{1-F(x)}\right)\right)$$

misalkan: $a = \log \beta_1$ dan $b = \frac{1}{\alpha_1}$

maka:

$$\log x = a + b\log\left(\log\left(\frac{1}{1-F(x)}\right)\right)$$

yang mana $F(x)$ dihampiri oleh $F_i = \frac{i-0,5}{n}, i = 1,2,...n$. Maka dapat dicari nilai α_1, β_1, dengan menggunakan data pada table (4.2) maka akan diperoleh perhitungan sebagai berikut yang disajikan pada table (4.4) dibawah ini:

Tabel 4.4 Hasil Perhitungan data *lifetimes of 8 Electronik*

i	x	$\log\left(\log\left(\frac{1}{1-\left(i-0,5/n\right)}\right)\right)$
1	0.03	-1.552
2	0.12	-1.044
3	0.22	-0.788
4	0.35	-0.602
5	0.73	-0.444
6	0.79	-0.296
7	1.25	-0.138
8	1.41	0.080
Jumlah	4.9	-4.787
Rata-rata	0.6125	-0.598

Dengan menggunakan rumus regresi sederhana maka diperoleh nilai b yang dapat ditunjukkan seperti dibawah ini:

$$b = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

$$= \frac{1,757631358}{1,89255}$$

$$= 0,928711$$

sehingga:

$$b = \frac{1}{\alpha_1}$$

maka:

$$\alpha_1 = \frac{1}{b} = \frac{1}{0{,}928711} = 1{,}07$$

dan,

$$a = \bar{y} - b\,\bar{x}$$

$$= -0{,}598 - (0{,}928 \times 0{,}6125)$$

$$= -1{,}257$$

sehingga: $a = \log \beta_1$

maka:

$$\beta_1 = e^a$$

$$= e^{-1{,}257} = 0{,}28$$

Kelompok data kedua digunakan untuk distribusi Weibull berparameter tiga, yaitu $\alpha_2, \beta_2, \gamma$, yang mana $\gamma = 1$. Selanjutnya berdasarkan persamaan (2.2.15) dan data pada tabel (4.3) akan dicari nilai awal untuk α_2, β_2, yang ditunjukkan seperti dibawah ini:

$$F(x) = 1 - e^{-(x-\gamma)^{\alpha} / \beta^{\alpha}}$$

$$e^{-(x-\gamma)^{\alpha} / \beta^{\alpha}} = 1 - F(x)$$

$$\log\left(e^{-\left(x-\gamma / \beta_2 \right)^{\alpha_2}} \right) = \log(1 - F(x))$$

$$-\alpha_2 \log\left(x-\gamma / \beta_2 \right) = \log(\log(1 - F(x)))$$

$$-\alpha_2 (\log(x-\gamma) - \log \beta_2) = \log(\log(1 - F(x)))$$

$$-\log(x-\gamma) = -\log \beta_2 + \frac{1}{\alpha_2} \log(\log(1 - F(x)))$$

$$\log(x-\gamma) = \log \beta_2 + \frac{1}{\alpha_2} \log\left(\log\left(\frac{1}{1 - F(x)} \right) \right)$$

misalkan: $a = \log \beta_2$ dan $b = \dfrac{1}{\alpha_2}$

maka:

$$\log(x-\gamma)=a+b\log\left(\log\left(\frac{1}{1-F(x)}\right)\right)$$

yang mana dimisalkan nilai $\gamma=1$ dan $F(x)$ dihampiri oleh $F_i=\frac{i-0{,}5}{n}, i=1,2,\ldots n$.

Maka dapat dicari nilai α_2,β_2, dengan menggunakan data pada table (4.3) sehingga diperoleh hasil perhitungan sebagai berikut:

Tabel 4.5 Hasil Perhitungan data *lifetimes of 8 Electronik*

i	x	$\log\left(\log\left(\frac{1}{1-\left(i-0{,}5/n\right)}\right)\right)$
1	1.25	-1.733218004
2	1.79	-1.23663231
3	1.8	-0.993715241
4	1.94	-0.82459744
5	2.38	-0.690114477
6	2.4	-0.574681204
7	2.87	-0.470032812
8	2.99	-0.370622279
9	3.14	-0.271554334
10	3.17	-0.166699071
11	4.72	-0.044268973
12	5.09	0.13994556
Jumlah	33.54	-7.236190585
Rata-rata	2.795	-0.603015882

Dengan cara yang sama menggunakan rumus regresi sederhana maka diperoleh nilai b yang dapat ditunjukkan seperti dibawah ini:

$$b=\frac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{\sum_{i=1}^{n}(x_i-\bar{x})^2}$$

$$=\frac{6{,}100048}{14{,}7223}$$

$$=0{,}4143$$

sehingga:

$$b = \frac{1}{\alpha_1}$$

maka:

$$\alpha_2 = \frac{1}{b} = \frac{1}{0,4143} = 2,413$$

dan,

$$\begin{aligned} a &= \bar{y} - b\bar{x} \\ &= -0.603015882 - (0.4143 \times 2.795) \\ &= -1,760 \end{aligned}$$

sehingga: $a = \log \beta_2$

maka:

$$\begin{aligned} \beta_2 &= e^a \\ &= e^{-1,760} = 0,172 \end{aligned}$$

sehingga diperoleh nilai awal untuk:

$$\alpha_1^0 = 1.07,\ \beta_1^0 = 0.28,\ \alpha_2^0 = 2.413,\ \beta_2^0 = 0.172,\ \gamma^0 = 1,\ w^0 = 0.4$$

berdasarkan persamaan (2.5.1) selanjutnya masukkan nilai awal diatas kedalam persamaan pada halaman IV-4 maka diperoleh:

$$f_1^0 = -304.224,\quad f_2^0 = 500.995,\quad f_3^0 = 16.769,$$

$$f_4^0 = 100.328,\quad f_5^0 = 35.695,\quad f_6^0 = 34.524$$

Nilai awal yang telah diperoleh selanjutnya akan dimasukkan kedalam matrik J seperti pada halaman IV-6 yang digunakan untuk menentukan akar persamaan dari data elektonik tersebut, dapat ditunjukkan sebagai berikut:

$$J = \begin{bmatrix} 12116.8 & -38135.6 & -14.6 & 146.9 & 80.4 & -6.6 \\ 1752.6 & -3861.3 & 21.4 & -484.2 & -192.0 & 14.6 \\ 19.2 & 21.4 & -46.1 & 3262.7 & -810.6 & -15.7 \\ 146.9 & -484.2 & 3262.7 & -9555.6 & 3185.4 & -218.5 \\ 80.4 & -192.0 & -810.6 & 3185.4 & 529.8 & -16.3 \\ -6.6 & 19.2 & 15.7 & -218.5 & -16.3 & -105.8 \end{bmatrix}$$

selanjutnya diperoleh:

$$J^{-1} = \begin{bmatrix} -0.00019373 & 0.00188178 & 0.00013321 & 0.00008606 & 0.00038215 & -0.00050388 \\ -0.00008778 & 0.00059781 & 0.00004267 & 0.00002775 & 0.00012350 & -0.00015966 \\ 0.00000148 & -0.00003616 & 0.00094950 & 0.00027103 & -0.00021054 & -0.00066300 \\ 0.00000066 & -0.00001847 & 0.00027018 & 0.00004192 & 0.00015001 & -0.00014721 \\ -0.00000424 & -0.000012250 & -0.00018839 & 0.00015744 & 0.00063657 & -0.00039352 \\ -0.00000438 & 0.00002631 & -0.00038901 & -0.00007107 & -0.00044053 & -0.00918301 \end{bmatrix}$$

- **Iterasi Pertama**. Berdasarkan persamaan (2.5.3) ditunjukkan sebagai berikut:

$$\alpha_1^1 = \alpha_1^0 - (b_{11}^0 f_1^0 + b_{12}^0 f_2^0 + \ldots + b_{16}^0 f_6^0)$$
$$= 0.056$$
$$\beta_1^1 = \beta_1^0 - (b_{21}^0 f_1^0 + b_{22}^0 f_2^0 + \ldots + b_{26}^0 f_6^0)$$
$$= -0.048$$
$$\alpha_2^1 = \alpha_2^0 - (b_{31}^0 f_1^0 + b_{32}^0 f_2^0 + \ldots + b_{36}^0 f_6^0)$$
$$= 2.418$$
$$\beta_2^1 = \beta_2^0 - (b_{41}^0 f_1^0 + b_{42}^0 f_2^0 + \ldots + b_{46}^0 f_6^0)$$
$$= 0.172$$
$$\gamma^1 = \gamma^0 - (b_{51}^0 f_1^0 + b_{52}^0 f_2^0 + \ldots + b_{56}^0 f_6^0)$$
$$= 0.983$$
$$w^1 = w^0 - (b_{61}^0 f_1^0 + b_{62}^0 f_2^0 + \ldots + b_{66}^0 f_6^0)$$
$$= 0.673$$

selanjutnya masukkan nilai diatas kedalam matrik jacobian seperti pada halaman IV-5 sehingga diperoleh:

$$f_1^1 = 283.667, \quad f_2^1 = -3.635, \quad f_3^1 = 75.629,$$
$$f_4^1 = -18.171, \quad f_5^1 = 42.314, \quad f_6^1 = 12.021$$

dan diperoleh juga nilai matrik Jacobian seperti pada halaman IV-6dengan memasukkan nilai $\alpha_1^1, \beta_1^1, \alpha_2^1, \beta_2^1, \gamma^1, w^1$ yang akan ditunjukkan sebagai berikut:

$$J = \begin{bmatrix} -5103.841 & -424.030 & -200.728 & -1818.095 & 276.507 & 32.623 \\ -132.737 & -101.918 & 6.342 & -15.391 & 5.694 & -0.368 \\ -200.728 & 6.342 & 787.774 & 6638.696 & -1760.534 & -83.973 \\ -1818.095 & -15.391 & 6638.696 & -33348.239 & 9060.872 & -267.954 \\ 276.507 & 5.694 & -1760.534 & 9060.872 & -2182.941 & 24.851 \\ 37.623 & -0,368 & -83.973 & -267.954 & 24.851 & -62.266 \end{bmatrix}$$

dan diperoleh:

$$J^{-1} = \begin{bmatrix} -0.0001 & 0.0004 & -0.0010 & -0.0001 & 0.0004 & 0.0019 \\ -0.0012 & -0.0046 & 0.0120 & 0.0008 & -0.0067 & -0.0230 \\ -0.0000 & 0.0001 & 0.0002 & 0.0000 & -0.0001 & -0.0003 \\ -0.0000 & 0.0001 & -0.0002 & 0.0002 & 0.0009 & -0.0002 \\ -0.0001 & 0.0003 & -0.0010 & 0.0008 & 0.0036 & -0.0007 \\ 0.0000 & -0.0001 & -0.0005 & -0.0006 & -0.0022 & -0.0136 \end{bmatrix}$$

yang mana misalakan J^{-1} merupakan nilai dari b

- Iterasi kedua

$$\alpha_1^2 = \alpha_1^1 - (b_{11}^1 f_1^1 + b_{12}^1 f_2^1 + \ldots + b_{16}^1 f_6^1)$$
$$= 0.120$$
$$\beta_1^2 = \beta_1^1 - (b_{21}^1 f_1^1 + b_{22}^1 f_2^1 + \ldots + b_{26}^1 f_6^1)$$
$$= -0.057$$
$$\alpha_2^2 = \alpha_2^1 - (b_{31}^1 f_1^1 + b_{32}^1 f_2^1 + \ldots + b_{36}^1 f_6^1)$$
$$= 2.411$$
$$\beta_2^2 = \beta_2^1 - (b_{41}^1 f_1^1 + b_{42}^1 f_2^1 + \ldots + b_{46}^1 f_6^1)$$
$$= 0.125$$
$$\gamma^2 = \gamma^1 - (b_{51}^1 f_1^1 + b_{52}^1 f_2^1 + \ldots + b_{56}^1 f_6^1)$$
$$= 0.799$$
$$w^2 = w^1 - (b_{61}^1 f_1^1 + b_{62}^1 f_2^1 + \ldots + b_{66}^1 f_6^1)$$
$$= 0.956$$

dengan cara yang sama maka diperoleh:

$$f_1^2 = 167.439, \quad f_2^2 = -13.918, \quad f_3^2 = -5.439,$$

$$f_4^2 = -30.391, \quad f_5^2 = 53.581, \quad f_6^2 = -3.666$$

ulangi kembali sampai k iterasi sehingga nilai $f_1, f_2 ... f_6 = 0$

BAB V

KESIMPULAN DAN SARAN

5.1 Kesimpulan

Kombinasi dua distribusi Weibull dengan menggunakan parameter campuran adalah mengkombinasikan dua buah distribusi Weibull yang berparameter dua dan tiga dengan parameter campuran w atau yang sering disebut distribusi Weibull campuran. Hasil kombinasi ini memiliki fungsi densitas peluang sebagai berikut:

$$f(x) = W\left(\alpha_1 \frac{x^{\alpha_1 - 1}}{\beta_1^{\alpha_1}}\right).e^{-\left(x/\beta_1\right)^{\alpha_1}} + (1-W)\left(\frac{\alpha_2 (x-\gamma)^{\alpha_2 - 1}}{\beta_2^{\alpha_2}}\right).e^{-\left(x/\beta_2\right)^{\alpha_2}}$$

Selanjutnya dengan menggunakan estimasi maksimum *likelihood* akan diperoleh nilai awal masing-masing parameter dengan memanfaatkan metode numerik yaitu metode Newton Raphson untuk menyelesaikannya.

Aplikasi *lifetimes of electronic components* menghasilkan nilai awal masing-masing parameter campuran untuk distribusi Weibull campuran adalah sebagai berikut:

$$\alpha_1^2 = 0.120, \beta_1^2 = -0.057, \alpha_2^2 = 2.411, \beta_2^2 = 0.125, \gamma = 0.799, w = 0.95$$

5.2 Saran

Tugas akhir ini membahas kombinasikan dua buah distribusi Weibull dengan menggunakan parameter campuran, yang mana distribusi Weibull yang dikombinasikan masing-masing berparameter dua dan tiga. Pembaca dapat mengkombinasikan distribusi yang lain sehingga menghasikan distribusi campuran. Selanjutnya dengan estimasi maksimum *likelihood* diperoleh nilai awal dari setiap parameter.

DAFTAR PUSTAKA

Danis, Wiliem dan Richard. " Matematical Statistic With Aplications. Edisi Pertama. USA. 2002.

Evans, Nicolas, dkk. "Statiscal Distributions". Edisi ketiga, halaman 3-6, 130-169. A Wiley –Interscience, Newyork. 2000.

Elisa dan John. Statistical methods For Survival Data Analysis. Edisi ketiga. Simulthaneouluc Canada, 2003.

Giessen, dan Justus. "The Weibull Distribution A Hand Book". Edisi pertama, halaaman 30, 197-130. Taylor and Francis Group-LLC, Newyork.2009.

Harinaldi. "Prinsip-prinsip Statistik untuk Teknik dan Sains". Edisi pertama, halaman 106. Erlangga, Jakarta. 2005.

Hines dan Douglas, "Probabilitas dan Statistik dalam Ilmu Rekayasa dan Manajemen", Edisi kedua, halaman 60-61, 184-185, 268. Universitas Indonesia. 1990.

Mahir, Ahmad Razali, dan Ali A. Salih, "Combining Two Weibull Distributions Using a Mixing Parameter", ISSN European Jurnal of Scientific Research. Vol.31, No.2, halaman 296-305, 2009.

Walpole, Ronald E dan Myers, Raymond H. "Ilmu Peluang dan Statistika untuk Ilmuwan dan Insinyur". Edisi keempat, halaman 76-77, 195-198. ITB Bandung. 1995.